# Fixed Income and Credit Risk
Fall 2012
Midterm Exam


| Professor | Assistant | Program |
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**Student's Family Name:**

**Student's First Name:**

**Date:**

**Ground Rules**

- **Closed book:** you are not allowed to use the material distributed during the course. This means that your notes, the lecture notes, the exercise sheets and associated solutions are not allowed, as well as outside books and lecture notes.

- You are allowed to use a calculator but not a programmable calculator or a laptop computer.

- The questions are independent and can be treated in the order of your choice. Just make sure to mention clearly which question you are solving.

- Make sure to write with an (black or blue) ink pen.

- The duration of the exam is 2 hours.

**Exercise N° 01.** **[1 point =** $0.25$ **points for each question.]**

Let us consider an amount of $A_t = 120000$ dollars (the principal) deposited at date $t$ in bank paying an annual nominal rate of $R = 5\%$.

*i*) If the annual nominal rate $R$ is compounded semi-annually, which is the amount of the principal after 8 years? Which is the associated effective annual rate?

*ii*) If the annual nominal rate $R$ is compounded quarterly, which is the amount of the principal after 8 years? Which is the associated effective annual rate?

*iii*) Compare the amount of the principal in *i*) and in *ii*) explaining the reason behind the observed difference.

*iv*) Determine, for both compounding frequency, the equivalent annual continuously compounded rate.

**Exercise N° 02.** **[1 point.]**

Calculate the duration $D$ and the convexity $\kappa$ of a bond paying (annually) a 8% annual coupon rate for 3 years (residual maturity), and the principal of 100 at the end of the third year. The price today (date $t = 0$) of the bond is denoted $CB(0, T)$. The yield to maturity (or the interest rate, assuming a flat term structure) is 10% (annual basis).

**Exercise N° 03.** **[1 point =** $0.3$ **points for** *i*) **and** *ii*)**,** $0.4$ **points for** *iii*)**.]**

Let us consider at the date $t = 0$ a coupon bond with a residual maturity of 2 years, paying (annually) a 6% annual coupon rate, and paying at the maturity date a principal of 100. The price today (date $t = 0$) of the bond is denoted $CB(0, T)$. The yield to maturity (or the interest rate, assuming a flat term structure) is 7% (annual basis). The duration and the convexity of this asset are respectively equal to $D = 1.9429$ and $\kappa = 5.0411$.

*i*) What is the relative bond price change, determined by means of its modified duration term only, if interest rates rise from 7% to 8% ?

*ii*) What is the approximated new bond price if we approximate the price variation taking into account also the convexity term ? Make a comparison with the approximation in point *i*) and with the true price variation.

*iii*) Repeat points *i*) and *ii*) assuming that now interest rates rise from 7% to 12%, and make a comment about the advantages and/or limits of the *"duration"* and *"duration + convexity"* approximations of bond price changes.

**Exercise N° 04. [1 point = 0.5 points for *i*), 0.5 points for *ii*).]**

*i*) What LIBOR and EURIBOR (for Euro) rates are ?

*ii*) Describe their differences.

**Exercise N° 05. [1 points.]**

Let us assume that at date $t$ a trader enters a Repo to take a long position on a coupon bond until date $T = t + n$. Both dates are between two coupon payments. Let us also denote the clean price at any given date $t$ by $CB_t^{clean}$, and the associated accrued interest by $AI_t$. The dirty price is $CB_t = CB_t^{clean} + AI_t$. Determine the relation between the coupon bond return and the annual repo rate $r$ (say) such that the profit of the trader is equal to zero. Provides also the associated conditions such that we have a positive and negative carry.

**Exercise N° 06. [1 points = 0.4 points for *i*), 0.3 points for *ii*), 0.3 points for *iii*).]**

Let us consider a financial market over a one-period only. This means that we consider only two dates: $t = 0$ and $t = 1$. At the date $t = 0$ two assets ($i \in \{0, 1\}$, say) are available in the market: the first one (the asset $i = 0$) is a risk-free bond maturing at $t = 1$ and with a price $S_0(0) = 1/3$. The second one (the asset $i = 1$) is a risky asset with price $S_1(0) = 1$. At date $t = 1$ we have $N = 3$ possible states of the world : $\omega_1$, $\omega_2$ and $\omega_3$. The payoff matrix of the 2 assets, at date $t = 1$, is the following $(2 \times 3)$-matrix $\mathbf{S}$ (say):

$$\mathbf{S} = \begin{bmatrix} S_0(1,\omega_1) & S_0(1,\omega_2) & S_0(1,\omega_3) \\ S_1(1,\omega_1) & S_1(1,\omega_2) & S_1(1,\omega_3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 6 \end{bmatrix},$$

where $S_i(t, \omega_j)$ denotes the price at date $t$ of the asset $i$ under the $j^{th}$ state of the world ($j \in \{1, 2, 3\}$).

*i*) Is this market arbitrage-free ? Motivate your answer on the basis of the existence of a vector of strictly positive state prices.

*ii*) Is this market complete ? Motivate your answer using the concept of positive state prices.

*iii*) Is there in the market $\{S(0), \mathbf{S}\}$ at least one equivalent martingale measure ? If yes, how many ? Motivate your answer building an equivalent martingale measure for the financial market $\{S(0), \mathbf{S}\}$ and using the risky asset pricing formula.