

## Fixed Income and Credit Risk: exercise sheet no. 10

Fall Semester 2010

1. A company has an outstanding five-year bond issue in USD paying an annual coupon of 4%. The bond currently is priced at par. The company wishes to issue a five-year bond in AUD, with the same seniority (and therefore the same expected recovery). Using the government discount curves provided in the spreadsheet, and assuming reasonable values for other needed parameters, find the annual coupon rate that the company would need to pay on its AUD issue in order for the AUD bond to also price at par. Repeat the exercise, but assuming that the USD bond currently trades at 95 or at 105. Are the USD and AUD yield spreads the same? Does the pricing of the USD bond impact the yield spread of the AUD issue?
2. In March 2009, in the so-called “CDS Big Bang”, the trading conventions for default swaps changed. Prior to this time, CDS typically traded with “zero upfront”, meaning that two parties in a CDS transaction would agree at the outset to a premium rate  $s^*$  such that the protection buyer and seller legs of the CDS were of equal value. The value of  $s^*$  was referred to as the quoted “fair spread”. In order to achieve better liquidity in the market, the Big Bang standardized the premiums that CDS would pay. Subsequently, all CDS contracts trade with a standard premium (either 100bp or 500bp). The protection buyer and seller thus must agree on an upfront payment at the outset of a CDS transaction. Under this new protocol, it is necessary to convert from the quoted fair spread to the required upfront payment. Derive this conversion. (This will require solving some equations numerically.) What assumptions are needed? How does the formula depend on which level of the standard premium is used?
3. For financial firms, there are often two classes of CDS: one that references senior debt, and another that references subordinated debt. Suppose that the five-year CDS referencing the senior unsecured debt of Bank of America currently trades at a fair spread of 60bp. What would be a reasonable range of fair spread quotes for a five-year CDS referencing subordinated debt of Bank of America?
4. Test the robustness of the Z-Score model using out-of-sample testing. In the spreadsheet, you will find the original 1968 dataset divided across two tabs: Training and Testing. This division has been performed randomly, with two-thirds of the sample in the training set, and one third in the testing set. Fit a simple linear discriminant model to the Training data under the following model specifications:
  - One univariate specification using each variable,
  - Bivariate specifications using the four variable pairs that were plotted in the lecture slides, and
  - The full specification using all five variables.

Assess the accuracy of each of these models on the Testing data. Compare these to the accuracy of the original Z-Score model (as applied only to the Testing data). Which model specification is most robust?

5. Can you find a non-linear transformation of one of the variables that improves the performance of either a bivariate or full model under the same criteria used above?
6. Build a numerical procedure (using either Excel or Matlab) to replicate the Merton model fitting on MSCI presented in the lecture slides. Examine the sensitivity of the overall default probability ( $P$  on the slides) and the annualized default probability ( $p$ ) to the assumption on  $T$  (the maturity of the debt). Provide some intuition for this behavior. Is this desirable for a credit model?
7. A first-to-default (FTD) is a simple CDO-like credit derivative. Consider a FTD as follows:
  - The reference portfolio contains equal exposure to three distinct firms. For each firm, the current fair spread for a five-year CDS is 100bp.
  - The FTD has maturity of five years, with annual payments.
  - The protection buyer pays a premium amount  $s$  each year on one unit of exposure as long as the contract is still active.
  - When (if) the first of the three firm defaults, the protection seller pays a loss amount on one unit of exposure. That is, the loss payment is  $(1 - R)$ . At this point, the contract is terminated.
  - If no defaults occur within five years, the contract terminates with the protection buyer having paid premium for five years, and the protection seller having paid nothing.

Work within the one-factor Gaussian copula framework to find the fair spread for the FTD. First, derive the fair spread under the special cases where  $\rho = 0$  and where  $\rho = 1$ . Second, build a numerical integration to solve for the fair spread where  $\rho = 0.3$ . What is the sensitivity of the fair spread to the correlation? Which side of the contract (the protection buyer or seller) can be said to be “long correlation”?