Fixed Income and Credit Risk : exercise sheet n° 06

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Professor	Assistant	Program
Fulvio Pegoraro	Roberto Marfè	MSc. Finance

Exercise N° 01 [Floating Rate Bond (from Veronesi (2010))].

At the date $t = 0.25$ we observe a floating rate bond maturing at $T = 6$ that pays a semiannual coupon (no spread). Last quarter the semiannually compounded rate was 3% (annual basis). At the same date we also observe in the market a coupon bond with price $CB(0.25, 0.5) = 100.0448$ and paying quarterly a coupon at the coupon rate of 3% (annual basis). What is the price of this floating rate bond ?

Exercise N° 02 [Floating Rate Bond (from Veronesi (2010))].

What is the price at $t = 0$ of a 0.5-year floating rate bond that pays a quarterly coupon equal to the floating plus a 1% spread (annual basis) ? We know the following:

a. We observe at $t = 0$ the price of a zero-coupon bond $B(0, 0.25) = 99.80$ (face value 100);

b. We observe a coupon bond paying an annualized rate of 2% quarterly with a price of $CB(0, 0.5) = 100.3960$;

Exercise N° 03 [Forward Rate Agreements with opposite payoffs (from Veronesi (2010), chapter 5)].

In the Example 3 of Lecture 6 we have seen the case of a firm asking to its bank to close at date $s = t + 3$months the FRA initiated at date t (notional amount $N = 1$). We have seen that the value was $V^{FRA}(s) = \$0.319638$. Show that the firm can achieve the same result by entering into a new FRA at time s with the reversed payoff.

Exercise N° 04 [Equivalence between Forward contract return and forward rate (from Veronesi (2010), chapter 5)].

Let us consider at date t a firm that enters into a forward contract with a bank to purchase six months later ($\tau = t + 0.5$ years) \$ 100 million-worth of 6-months Treasury bills for a price $\Phi(t, \tau, T)$ specified today (for \$ 100 par value).

i) What purchase price would the bank quote to the firm, for the 6-months T-bill, given that $B(t, t + 0.5\,\text{years}) = 0.97728$ and $B(t, t + 1\,\text{year}) = 0.95713$?

ii) Assuming that the firm enter into a forward contract to purchase $M = 1.02105$ million of 6-month T-bill on τ (with \$100 of par value, and $B(t + 0.5\,\text{years}, t + 1\,\text{year}) = \$98.89$), show that the forward contract yields a return on investment equal to the date-t forward rate over the period (τ, T) of the Example 1 in Lecture 6.

Exercise N° 05 [Forward on a Coupon Bond].

Let us suppose that an investor is interested to buy at date t a forward contract for delivery at date $T > t$ of a coupon bond with maturity date $\widetilde{T}$. Assuming that the date-t price of the ZCB with maturity date T is $B(t, T) = 97$ and that today the coupon bond has a price $CB(t, \widetilde{T}) = 115$, determine the forward price.

Exercise N° 06 [Value of a Swap (from Veronesi (2010))].

Determine the value of a 1.5-year swap, with swap rate $c = 5.52\%$ and notional $N = 100$ million. Use the following discount factors:

$$
\begin{aligned}
& [B(0, 0.25), B(0, 0.5), B(0, 0.75), B(0, 1), B(0, 1.25), B(0, 1.5), B(0, 1.75), B(0, 2)] \\
= \; & (0.9848, 0.9745, 0.9618, 0.9490, 0.9353, 0.9215, 0.9084, 0.8953)\,.
\end{aligned}
$$

You are told that this is a swap at initiation. Is the value accurate (remember that the swap's fixed lag is paid semiannually, not quarterly)?

Exercise N° 07 [Exercise N° 06, continued (from Veronesi (2010))].

Let us consider the same swap as in the previous exercise. what is the value of the swap three months after the initiation, where the discount factors are now:

$$
\begin{aligned}
& [B(0.25, 0.5), B(0.25, 0.75), B(0.25, 1), B(0.25, 1.25), B(0.25, 1.5), B(0.25, 1.75), B(0.25, 2), B(0.25, 2.25)] \\
= \; & (0.9840, 0.9680, 0.9520, 0.9360, 0.9190, 0.9040, 0.8880, 0.8730)\,,
\end{aligned}
$$

and given that the 6-month LIBOR rate at date $t = 0$ was $r_2(0) = 5.170\%$ (annual basis).

Exercise N° 08 [Defaultable ZCB price in a simple setting with payment at maturity].

Let us consider the problem to price at date $t = 0$ a defaultable ZCB with maturity date T and constant recovery rate RR. If the default date τ has not appeared before T, the payoff is equal to one, otherwise if $\tau \leq T$, then the bondholder receives RR at T. Let us assume to have a deterministic risk-free rate r_t over the time horizon $[0, T]$, and let us assume that the non-negative random variable τ has an historical cumulative probability function $F(t) := \mathbb{P}(\tau < t)$, with $F(t) < 1$ for any $t < T$. Under the risk-neutral probability measure we have $F^*(t) := \mathbb{Q}(\tau < t)$.

Show that :

a) the price at date $t = 0$ (the initial date) of the defaultable ZCB $DB(0,T)$ is given by:

$$\begin{aligned} DB(0,T) &= E_0^{\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{T-1} r_i\right)\left(\mathbb{I}_{\{\tau>T\}} + RR \times \mathbb{I}_{\{\tau\leq T\}}\right)\right] \\ &= B(0,T) - (1-RR)\exp\left(-\sum_{i=0}^{T-1} r_i\right) F^*(T) \end{aligned}$$

b) the price at date $t \in (0,T)$, conditionally to $\tau > t$ (the *predefault price*), is given by:

$$\begin{aligned} DB(t,T) &= E_t^{\mathbb{Q}}\left[\exp\left(-\sum_{i=t}^{T-1} r_i\right)\left(\mathbb{I}_{\{\tau>T\}} + RR \times \mathbb{I}_{\{\tau\leq T\}}\right) \mid \tau > t\right] \\ &= B(t,T) - (1-RR)\exp\left(-\sum_{i=t}^{T-1} r_i\right) \frac{F^*(T) - F^*(t)}{1-F^*(t)}\,. \end{aligned}$$

In addition : *c*) provide an interpretation of $\dfrac{F^*(T) - F^*(t)}{1-F^*(t)}$; *d*) provide the defaultable ZCB pricing formula at date $t \in (0,T)$, regardless the fact the $\tau > t$ or not. Let us denote this price $\widetilde{DB}(t,T)$.

Exercise N° 09 [Exercise N° 08, continued].

Let us consider again the pricing problem presented in Exercise N° 06, but let us assume now to be in a continuous time setting with a recovery rate function of the default date, $RR(\tau)$ say. Let us denote the historical and risk-neutral probability density function (p.d.f.) of the random default date τ by $f(t)$ and $f^*(t)$, respectively. Write the pricing formulas: *i*) $DB(0,T)$; *ii*) $DB(t,T)$, and *iii*) $\widetilde{DB}(t,T)$.

Exercise N° 10 [Defaultable ZCB price in a simple setting, intensity function and credit spread].

Let us consider the defaultable ZCB pricing problem, in continuous time, presented in Exercise N° 07. Let us assume to have a deterministic risk-free rate r_t over the time horizon $[0,T]$, and let us assume that the non-negative random variable τ has an historical cumulative probability function $F(t) := \mathbb{P}(\tau < t)$, with $F(t) < 1$ for any $t < T$. Under the risk-neutral probability measure we have $F^*(t) := \mathbb{Q}(\tau < t)$. The historical and risk-neutral probability density function (p.d.f.) of the random default date τ is denoted $f(t)$ and $f^*(t)$, respectively.

Let us introduce the *hazard function* Λ defined by:

$$\Lambda(t) = -\log(1 - F(t))\,,$$

and its derivative $\lambda(t) = \dfrac{f(t)}{1-F(t)}$ is called *hazard rate* (or, intensity rate).

Represents the default probability $\mathbb{P}(\tau > t)$ and the predefault price $DB(t,T)$ in terms of $\lambda(t)$. Provide also an interpretation of $\lambda(t)$ in terms of default probability. Moreover, write the credit spread assuming that the recovery and intensity rates are constant.

Exercise N° 11 [Defaultable ZCB price in a simple setting with payment at hit].

Let us consider the defaultable ZCB pricing problem, in continuous time, presented in Exercise N° 03. Nevertheless, in case of default at time $\tau \leq T$, the recovery rate $RR(\tau)$ is paid at τ and not at T. Write the pricing formulas: *i)* $DB(0,T)$; *ii)* $DB(t,T)$, and *iii)* $\widetilde{DB}(t,T)$.

Exercise N° 12 [Defaultable ZCB pricing formula in discrete-time with zero recovery].

Let us consider a no-arbitrage discrete-time setting and the problem to price at date t a defaultable ZCB, with maturity date at $t+h$, and issued by the firm i. Prove that:

$$\begin{aligned} DB_i(t,t+h) &= E[M_{t,t+1} ... M_{t+h-1,h} \mathbb{I}_{\tau_i > t+h} \,|\, I_t] \\ &= E[M_{t,t+1} ... M_{t+h-1,h} \exp(-\lambda^i_{t+1} - ... - \lambda^i_{t+h}) \,|\, I_t] \,, \end{aligned}$$

where $I_t = (\underline{x}_t, \underline{x}^i_t, \tau_i > t)$ and where λ^i_{t+1} denotes the one-period conditional survivor intensity over $(t, t+1)$:

$$\begin{aligned} \mathcal{S}^i_{\tau,t,t+1} &= \mathbb{P}[\tau_i > t+1 | \tau_i > t, \underline{x}, \underline{x^j}, j = 1, \ldots, n] \\ &= \mathbb{P}[\tau_i > t+1 | \tau_i > t, x_{t+1}, x^i_{t+1}] \\ &= \exp(-\lambda^i_{t+1}), \text{ say}, \forall t \,. \end{aligned}$$