Fixed Income and Credit Risk : exercise sheet n° 01

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Professor	Assistant	Program
Fulvio Pegoraro	Roberto Marfè	MSc. Finance

Exercise N° 01.

Let us consider an amount of $P_t = 25000$ dollars (the principal) deposited at date t in bank paying an annual nominal rate of 3% compounded quarterly. Which is the amount of the principal after 5 years and associated effective annual rate? And if the annual nominal rate is compounded monthly? Determine, for both compounding frequency, the equivalent annual continuously compounded rate.

Exercise N° 02.

Let us assume that at date t we observe in the bond market the following ZCB prices : $B(t, t+0.5) = 0.99$, $B(t, t+1) = 0.985$, $B(t, t+1.5) = 0.98$ and $B(t, t+2) = 0.975$. What is the price of a coupon bond with residual maturity 2 years, face value 100 dollars, and annual coupon rate of 4% with semiannual payments?

Exercise N° 03.

What is the price of a coupon bond with residual maturity 3 years, face value 100 dollars, a yield to maturity 5% (with annual compounding), a 10% annual coupon rate with annual coupon frequency? How the coupon bond price changes if the yield to maturity increases to 7%, 9% and 10%?

Exercise N° 04.

A bullet bond priced at 103 pays annual coupons of 7% of a principal of 100 for the next 4 years. The bond pays the principal at the end of the fourth year. What is the annual yield to maturity?

Exercise N° 05.

Let us consider at date $t = 0$ a bullet bond with coupon rate c on a face value C_T paid at $T > 0$, price $CB(0, T)$ and annually compounded yield to maturity $Y^{CB}(0, T) = Y$. The price-yield relation is:

$$CB(t, T) = \sum_{i=1}^{T} (c \times C_T) \times (1 + Y)^{-i} + \frac{C_T}{(1 + Y)^T} .$$

What happens to the price $CB(t, T)$ when $c = Y$? And when $c \neq Y$?

Exercise N° 06.

Let us imagine to have the five bonds with a principal of 100 and paying a constant coupon [see Table 1].

Table 1

	Residual Maturity (years)	Annual Coupon Rate (%)	Price
Bond A	1	5.00	100.91
Bond B	2	6.00	103.02
Bond C	3	7.50	107.54
Bond D	4	5.25	101.18
Bond E	5	8.00	112.72

a) Determine the associated annually compounded yield to maturities.

b) Extract from the coupon bond prices in Table 1 the annually compounded discount rates (i.e., ZCB yield to maturity) $Y(t, t+i)$, $i \in \{1y, \ldots, 5y\}$, corresponding to cash flows from periods $\{t + 1y, \ldots, t + 5y\}$.

c) Let us imagine to introduce in the market a new bond B' (say) with residual maturity of 2 years, a principal of 100, an annual coupon rate of 12% and with price $CB_{B'}(t, t+2y) = 114.29$. Determine the associated annually compounded yield to maturity. Is the coupon bond price arbitrage-free ?

d) Suppose that the market price of the bond is $CB_{B'}(t, t+2y) = 115.00$. Propose a transaction providing an arbitrage opportunity assuming that in the market are traded ZCB $B(t, t+i)$, $i \in \{1y, 2y\}$.

e) Now propose another transaction, providing a risk-free profit, assuming that only the bonds in Table 1 are traded.

Exercise N° 07.

Let us consider at date t, a bullet bond with $C_T = 100$, an annual coupon rate of 7% and time-to-maturity 3 years ($T - t = 3$ years). Suppose that, at date t, ZCBs with face value equal to 1 dollar, are traded with residual maturity of 1, 2 and 3 years. Assume that the market prices are $B(t, t + 1y) = 0.98$, $B(t, t + 2y) = 0.94$ and $B(t, t + 3y) = 0.90$. Let us imagine that at date t the market price that bullet bond at $CB(t, t + 3y) = 112$. Is there an arbitrage opportunity? Why?

Exercise N° 08.

Let us consider at date t a bond market were are traded two (unitary face value) zero-coupon bonds with maturity dates τ and T, respectively ($t < \tau < T$). The associated prices are $B(t, \tau)$ and $B(t, T)$. At the same date t we have a forward contract with settlement date τ and maturity date T. This contract provides, in return for an investment of 1 unit of money at date τ, an amount of $\exp[(T - \tau) \times R(t, \tau, T)]$ at time T; $R(t, \tau, T)$ indicates the continuously compounded forward rate prevailing at date t.
Let us imagine that, at date t, we have:

$$R(t, \tau, T) > \frac{1}{(T - \tau)} \ln \left(\frac{B(t, \tau)}{B(t, T)} \right) .$$

Show that there is an arbitrage opportunity.

Exercise N° 09.

Let us consider at date t the 2-year ZCB with price $B(t, t+2y) = 97$ and the 5-year ZCB with price $B(t, t+5y) = 93$ (both with unitary face value). What is the continuously compounded forward rate $R(t, t+2y, t+5y)$? And the annually compounded one?

Exercise N° 10.

A TIPS was issued on April 15, 1996, with an annual coupon rate of 3.5 %. The first coupon payment date for this asset was October 15, 1996. We have that $CPI^{ref}(15/04/1996) = 120.00$ while $CPI^{ref}(15/10/1996) = 135.00$ (inflationary period). If the par value is $C_T = \$1$ million, what was the coupon payment on October 15, 1996 ?

Exercise N° 11.

On the settlement date of May 14, 2007, a trader wanted to finance a 10 million dollars par amount of a 6.375% (annual basis), August 15, 2027 (maturity date), Treasury bond overnight (i.e., one day). The coupons are paid each six months, and the bond must have accrued 88 days of interest (the difference between the last coupon date of February 15, 2007 and the settlement date). The difference between the next coupon date, August 15, 2007, and the last coupon date is 181 days. Let us imagine that the clean price of the bond, at the current and the following date, is $CB^{clean} = 118.842$, and let us assume $C_T = 100$. The prevailing overnight rate was 6% (annual basis). At what price should the repo dealer sell the bond on May 15, 2007, so as to earn a repo rate of $r = 6\%$? What happens to the profit of the trader if the coupon rate is 9% (annual basis) instead of 6.375%?

Exercise N° 12.

Let us assume that at date t a trader enters a Repo to take a long position on a coupon bond until date $T = t + n$. Both dates are between two coupon payments. Let us also denote the clean price at any given date t by CB_t^{clean}, and the associated accrued interest by AI_t. The dirty price is $CB_t = CB_t^{clean} + AI_t$. Determine the relation between the coupon bond return and the annual repo rate r (say) such that the profit of the trader is equal to zero. Provides also the associated conditions such that we have a positive and negative carry.