Fixed Income and Credit Risk
Winter 2013
Exam

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Student's Family Name:

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Date:

Ground Rules

- **Closed book:** you are not allowed to use the material distributed during the course. This means that your notes, the lecture notes, the exercise sheets and associated solutions are not allowed, as well as outside books and lecture notes.

- You are allowed to use a calculator but not a programmable calculator or a laptop computer.

- Make sure to write with an (black or blue) ink pen.

- The duration of the exam is 2 hours.

Exercise N° 01 **[1 point = 0.3 points for *i*), 0.4 points for *ii*), 0.3 points for *iii*)].**

Answer the following questions:

i) Describe the Repo and Reverse Repo contracts highlighting the main features.

ii) What is the return on capital for a trader who entered into a one-month repo where $P_t = 98.5$, $P_T = 99.01$, repo rate = 5% and haircut = 0.8 ?

iii) What is the profit for a trader who entered into a one-week reverse repo where $P_t = 99.40$, $P_T = 99.48$ and repo rate = 6% (haircut = 0) ?

Exercise N° 02 **[1 point = 0.5 points for *i*), 0.5 points for *ii*)].**

i) Let us assume that the LOP holds. Show that, in an incomplete market without redundant assets ($k = rank(\mathbf{S}') = d + 1 < N$) any payoff $y \in \mathcal{M}(\mathbf{S})$ can be priced by the following formula: $q(y) = y' \, R^{(S)} \, S(0) = S(0)' \, L^{(S')} \, y$.

ii) Then, let us consider an incomplete market with redundant securities ($k < d + 1$, $k < N$). Let us denote with $\bar{\mathbf{S}}$ the (k, N) payoff matrix of the no redundant assets and with $\bar{S}(0)$ the vector of these k asset prices. Show that any payoff $y \in \mathcal{M}(\mathbf{S})$ can be priced by the following formula: $q(y) = y' \, R^{(\bar{S})} \, \bar{S}(0) = \bar{S}(0)' \, L^{(\bar{S}')} \, y$.

Exercise N° 03 [1 point].

Let us consider a discrete-time univariate Gaussian term structure model, in which the factor x_{t+1} has an historical dynamics described by the Gaussian AR(p) process:

$$\begin{aligned} x_{t+1} &= \nu + \varphi_1 x_t + \ldots + \varphi_p x_{t+1-p} + \sigma\varepsilon_{t+1} \\ &= \nu + \varphi' X_t + \sigma\varepsilon_{t+1} \,, \end{aligned}$$

where ε_{t+1} is a Gaussian white noise with $\mathcal{N}(0,1)$ distribution. We have: $\varphi = [\varphi_1, \ldots, \varphi_p]'$, $X_t = [\, x_t, \ldots, x_{t+1-p}\,]'$, and where $\sigma > 0$, ν and φ_i, for $i \in \{1, \ldots, p\}$, are scalar coefficients. Let us also assume that the stochastic discount factor (SDF) $M_{t,t+1}$ for the period $(t, t+1)$ has the following exponential-affine specification:

$$M_{t,t+1} = \exp\left[-\beta - \alpha' X_t + \Gamma_t \varepsilon_{t+1} - \frac{1}{2}\Gamma_t^2\right] .$$

Prove that the price at date t of the zero-coupon bond with time to maturity h is :

$$B(t, t+h) = \exp(c_h' X_t + d_h) \,, \;\; h \geq 1 \,,$$

where c_h and d_h satisfies the recursive equations :

$$\begin{aligned} c_h &= -\alpha + \Phi' c_{h-1} + c_{1,h-1}\sigma\gamma = -\alpha + \Phi^{*'} c_{h-1} \,, \\ d_h &= -\beta + c_{1,h-1}(\nu + \gamma_o\sigma) + \tfrac{1}{2} c_{1,h-1}^2 \sigma^2 + d_{h-1} \,, \end{aligned}$$

with :

$$\Phi^* = \begin{bmatrix} \varphi_1 + \sigma\gamma_1 & \dots & \dots & \varphi_{p-1} + \sigma\gamma_{p-1} & \varphi_p + \sigma\gamma_p \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & 1 & 0 \end{bmatrix}.$$

and where the initial conditions are $c_0 = 0, d_0 = 0$ (or $c_1 = -\alpha, d_1 = -\beta$); $c_{1,h}$ is the first component of the p-dimensional vector c_h.

Exercise N° 04 [1 point].

What is the price at $t = 0$ of a 0.5-year floating rate bond that pays a quarterly coupon equal to the floating plus a 1.5% spread (annual basis) ? We know the following:

a. We observe at $t = 0$ the price of a zero-coupon bond $B(0, 0.25) = 98.80$ (face value 100);

b. We observe a coupon bond paying an annualized rate of 2.5% quarterly with a price of $CB(0, 0.5) = 100.960$;

Exercise N° 05 [1 point].

Determine the value of a 1.5-year swap, with swap rate $c = 5.52\%$ and notional $N = 100$ million. Use the following discount factors:

$$\begin{aligned} & [B(0,0.25), B(0,0.5), B(0,0.75), B(0,1), B(0,1.25), B(0,1.5), B(0,1.75), B(0,2)] \\ = \quad & (0.948, 0.945, 0.918, 0.990, 0.953, 0.915, 0.984, 0.853)\,. \end{aligned}$$

You are told that this is a swap at initiation. Is the swap rate correct (remember that the swap's fixed lag is paid semiannually, not quarterly)? If not, which is the correct one ?

Exercise N° 06 [1 point].

Derive the price of a bond backed by a homogeneous pool of credits. The pricing expression may be left in terms of integrals or equations to be evaluated numerically. Be clear about the parameters or market data you need to assume.

The bond is specified as follows:

- The bond has a maturity of five years, with annual coupon payments.
- The pool consists of a large number of credits, each having a five-year CDS spread of s. The credits are equally weighted in the pool. The total face value of the pool is equal to the original notional of the bond.

- The notional of the bond begins at 100%, and is written down over time according to default losses on the pool, based on a threshold $a = 50\%$. At time t from the issue of the bond, if cumulative losses on the pool are less than a, then the bond notional is still 100%. If pool losses are greater than a, then the bond notional is reduced by the excess of pool losses over a.
- At the end of each year, a fixed coupon c is paid based on the bond notional value at the beginning of the year.
- At bond maturity, the (possibly reduced) notional is paid back.