Measuring Macroeconomic Tail Risk

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Abstract

This paper proposes a predictive approach to estimate macroeconomic tail risk dynamics over the long run (1876-2015). Our approach circumvents the scarcity of large macroeconomic crises by using observable predictive variables in a large international panel. This method does not require asset price information, which allows us to evaluate the empirical validity of rare disasters models. Our macro risk estimates covary with asset prices and forecast future stock returns, in line with the prediction that macroeconomic tail risk drives the equity premium. A rare disaster model, calibrated from macroeconomic data alone, further supports this interpretation.

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Many puzzles in macro-finance arise from the inability of models to reconcile, quantitatively, asset prices and macroeconomic risk. A classic example is the equity premium puzzle: averaged stock returns are far too high to be explained by the observed risk in consumption (Mehra and Prescott, 1985). Another well-known example is that valuation ratios, such as the dividend-price ratio, can forecast stock returns (Campbell and Shiller 1988b). Likewise, stock market volatility is too high to reflect forecasts of future dividends (Shiller, 1981). A potential resolution of these puzzles is that we do not observe the risk that agents genuinely care about. Rare disaster models (Rietz, 1988; Barro, 2006), in particular, posit that agents care about infrequent but severe macroeconomic crises. Gabaix (2012), Gourio (2012), and Wachter (2013) show theoretically that variation in macroeconomic tail risk can rationalize the above puzzles. Unfortunately, the success of rare disaster models relies on an elusive state variable: the time-varying probability of a large macroeconomic crisis. The apparent difficulty in measuring macroeconomic tail risk has created controversy over whether rare disaster models are falsifiable.\footnote{For instance, John Campbell referred to the probability of rare disasters as economists’ “dark matter” in his 2008 Princeton Lectures in Finance (Campbell, 2017).}

In this paper, we propose a straightforward method to estimate time-varying disaster probabilities based on predictive regressions. Our approach does not require asset price information and can be deployed over a very long timeframe. It circumvents the scarcity of large macroeconomic crises by exploiting the informational content of variables that forecast such crises and, in the spirit of Barro (2006), by using a broad cross-section of countries. Our measure of disaster risk allows us to document for the first time that stock prices actually respond to macroeconomic tail risk—arguably the main testable prediction of rare disaster models.

Concretely, we set up a probit panel model where the left-hand-side variable is an indicator equal to one when a given country \(i\) experiences a large macroeconomic crisis in year \(t+1\), and zero otherwise. Our international panel builds on Barro and Ursúa (2008) and covers 1876 to 2015, comprising 42 countries. Our baseline specification defines a macroeconomic crisis as a two-standard-deviation drop in consumption below its long-term growth path in a given year. On the right-hand side of our forecasting model, we consider a rich set of time-\(t\) predictor variables, including macroeconomic variables such as realized crises at home and abroad, but also wars, political crises, natural disasters, and (potentially) asset prices. The fitted values in this regression yield the time-\(t\) probability of a macroeconomic crisis in country \(i\) in year \(t+1\).

Figure 1 gives an overview of our main result. It shows the next-year probability of a macroeconomic crisis in the United States. The probability, which we denote by \(\hat{\pi}\), mostly varies between 0% and 10%, with higher levels during the Depression of 1893, the Great Depression, the two world wars, and the Great Recession of 2009. Our forecasting model performs remarkably...
well out-of-sample and could, therefore, be used to predict crises in real-time. The most reliable predictors include realized crises at home and in neighboring countries, wars, and the U.S. (or world) dividend-price ratio. Our macroeconomic tail risk (or macro risk, for short) estimates are remarkably insensitive to the sample period and econometric specification. Probit coefficients are broadly stable over time and between advanced and emerging countries. Changing the crisis cutoff to 1.5, 2.5, and 3 standard deviations or using the crises identified in Barro and Ursúa (2008) also produce qualitatively similar results.

Under the condition that our predictor set approximately spans the information set of economic agents, \( \hat{\pi} \) can be interpreted as the rational expectation probability of a large macroeconomic crisis. It thus provides a benchmark with which to evaluate theories where tail risk plays a role. In the second part of the paper, we ask whether macro risk is related to the equity premium. We find that asset prices tend to be low (high dividend-price ratio) when \( \hat{\pi} \) is high (corr. = 0.50). By itself, the U.S. dividend-price ratio—a standard equity premium proxy in the literature (e.g., van Binsbergen and Koijen, 2010)—captures a lot of information about future crises. A one standard deviation increase in the D/P ratio raises the likelihood of a crisis by 2.1 percent, which is about half the unconditional crisis probability of 3.9%. This is precisely what one would expect if investors were shunning stocks when tail risk is high, as predicted by rare disasters models. We find similar results using the equity premium proxy proposed by Martin (2017). Furthermore, macro risk directly forecasts international stock returns, further confirming the link between tail risk and the equity premium. In contrast, macro risk is only weakly related to future consumption growth, meaning that our estimates reproduce the well-known disconnect between the equity premium and consumption. Finally, these results hold when we construct macro risk excluding asset-price predictors.

We next ask if rare disaster models help rationalize the equity premium puzzle and other patterns we observe in the data. Following the Mehra and Prescott (1985) approach, we calibrate the stochastic process for consumption dynamics, derive the process for prices, and compare those prices to their empirical counterparts. A benefit of our approach is that we can work with estimated consumption dynamics that exclude asset-price predictors, meaning that consumption parameters are not reverse-engineered to fit the asset pricing data.

We consider a rare disaster model similar to Wachter (2013), in which agents have recursive preferences. Consumption growth follows an exogenous stream, which is subject to rare disasters that occur with a time-varying probability.\(^2\) The model is in discrete-time and is of

\(^2\)Another strand of research maintains a constant disaster probability but assumes that the representative agent learns about the model parameters or states (e.g., Weitzman, 2007; Koulovatianos and Wieland, 2011; Du and Elkamhi, 2012; Orlik and Veldkamp, 2014; Johannes et al., 2016). Martin (2013) shows that constant disaster risk spreads across assets in a multiple-tree economy and leads to endogenous risk premia dynamics.
the exponential affine type so that it can be solved with standard techniques and yields simple, closed-form solutions. We use this model as a laboratory to assess both the qualitative and quantitative effects of our measure of time-varying disaster probability—that is, our model’s only state variable. The model generates a high and volatile equity premium and a low risk-free rate under conservative preferences. It can also reproduce both the predictability of stock returns and the lack of predictability of consumption growth by asset prices that we observe in the data. Alternative model specifications support the idea that time-varying disaster probability is a priced state-variable, responsible for a sizable component of the equity premium and its dynamics—even when its endogenous correlation with the D/P ratio matches that in actual data. Taken together, our findings suggest macroeconomic tail risk can rationalize the equity premium and risk-free rate puzzles, the excess volatility puzzle, and the predictability of aggregate stock market returns by the dividend-price ratio.

Our paper is related to a large body of work in asset pricing and macroeconomics that is concerned with measuring the variability of output growth (e.g., DeLong and Summers, 1986; Stock and Watson, 2002; Schorfheide et al., 2018) and macroeconomic uncertainty (e.g., Orlik and Veldkamp, 2014; Jurado et al., 2015; Nakamura et al., 2017). Giglio et al. (2016) and Adrian et al. (2019) relate the conditional distribution of GDP growth to measures of financial market distress. In a seminal paper, Barro (2006) documented that in international consumption data, disaster risk is compatible with a high equity premium. Barro further conjectured that variation in disaster risk could rationalize a volatile equity premium. We provide evidence supporting Barro’s conjecture. A number of recent works provide indirect measures of macroeconomic tail risk. Berkman et al. (2011) use the number and severity of international political crises to proxy for time-varying disaster risk, Colacito et al. (2016) use survey data, and Manela and Moreira (2017) construct a volatility index using front-page articles of the Wall Street Journal, which they relate to disaster concerns. These papers rely on the assumption that the proposed proxy adequately reflects macroeconomic concerns. In contrast, our approach takes candidate proxies as input and measures tail risk by projecting them on future tail events. The fact that our risk estimate varies over time is the direct consequence that these candidate proxies forecast future macroeconomic crises.

Another suitable vehicle for estimating tail risk is option prices. Our macro risk measure corresponds to the probability of a macroeconomic crisis under rational expectation, provided our predictor set spans the information set of economic agents. Interpreting option prices, in contrast, does not need assumptions about investors’ information set. Both Siriwardane (2015) and Barro and Liao (2019) back out the probability of a macroeconomic crisis, the former under
the risk-neutral measure and the latter under the physical measure, over the shorter period over which option data is available. Doing so requires assumptions about investors’ preferences and the joint behavior of the stock markets and macroeconomic outcomes. In practice, the option-based estimates of Siriwardane (2015) and Barro and Liao (2019) are tightly linked to volatility indices such as the VIX. Their estimates can thus be cast within our predictive framework as one among many candidate predictors for future crises. We find that stock volatility strongly forecasts macroeconomic crises but that many other variables have incremental forecasting power.

The rest of this paper is organized as follows. Section I presents our data and methodology. Section II documents that large macroeconomic crises are forecastable. Section III presents estimated macro risk and studies the relationship between macro risk and variables such as stock returns and macroeconomic growth. Section IV calibrates a rare disaster model using macro risk dynamics. We conclude in Section V. An Online Appendix contains additional results and proofs.

I. Methodology and Data

We begin by presenting our econometric framework. We are interested in quantifying macroeconomic tail risk in country \( i = 1, \ldots, N \), denoted by \( \pi_{i,t} \), and defined as the year \( t \) probability of a macroeconomic crisis at time \( t + 1 \),

\[
\pi_{i,t} \equiv \Pr(\text{Crisis}_{i,t+1} = 1 | I_{i,t}),
\]  

(1)

where the probability \( \Pr(., | I_{i,t}) \) is taken with respect to information \( I_{i,t} \) available to economic agents at time \( t \).

We define a macroeconomic crisis as a severe decline in consumption or output during a given year. A country \( i \) experiences a macroeconomic crisis in time \( t \) if the log growth rate in country \( i \), \( \Delta c_{i,t} \), falls by \( k \) standard deviations (SD) from its long-term growth path:

\[
\text{Crisis}_{i,t} = \begin{cases} 
1, & \text{if } \Delta c_{i,t} < \text{mean}(\Delta c_{i,t}) - k \times \text{SD}(\Delta c_{i,t}). \\
0, & \text{otherwise}.
\end{cases}
\]

(2)

This modeling choice conforms with the common approach in the rare disasters literature to define tail events as cumulative declines in consumption (or GDP) that exceed a fixed cutoff value. An alternative choice, pursued in Adrian et al. (2019), consists in constructing crisis
probabilities from quantile regressions. We show in the Online Appendix that this approach yields similar tail risk estimates.

The fixed-cutoff assumption amounts to treating the long-term growth rate and volatility in normal times as constant parameters, a common assumption in the rare disasters literature. Variation in $\pi_{i,t}$ thus summarizes movements in recession risk, which depends on both the level, volatility, but also on higher moments of the growth process. Our baseline crisis definition departs from the rare disasters literature in that macroeconomic crises are one-year events. Although it is natural to think of macroeconomic crises as events that unfold over several years, measuring macroeconomic tail risk in calendar time is better suited to our forecasting framework.\footnote{Defining crises as one-year events is also consistent with the assumption in the model in Section IV that macroeconomic disasters occur instantaneously. Our modeling approach is thus immune to the measurement issues highlighted by Constantinides (2008), Donaldson and Mehra (2008), and Julliard and Ghosh (2012).} We nevertheless explore crisis probabilities using crises identified in Barro and Ursúa (2008) in Section III.B.

A. Empirical implementation

We need to introduce a few assumptions to operationalize the approach in Eq. (2) and (3). We are interested in severe and rare crises, which are approximately comparable across countries. Our baseline specification thus sets the crisis cutoff $k = 2$ in Eq. (2). We compute the long-term growth rate and standard deviation on a country by country basis. This ensures that over the long run, crises are approximately equally rare in the cross section. For standard deviations to reflect macroeconomic volatility in good times, we use series winsorized at the 2.5% level in both tails. Further, as noted in, e.g., Nakamura et al. (2013), the series assembled by Barro and Ursúa (2008) are in many countries much less volatile after Word War II. To a large extent, this fall in volatility likely reflects changes in national accounts measurement around this time (Romer, 1986; Balke and Gordon, 1989). We thus identify macroeconomic crises using distinct standard deviations for the pre- and post-WW2 samples in our baseline specification.\footnote{Some countries have very few observations in the prewar period. We use the full-sample standard deviation whenever we have less than 25 years of prewar data.}

We find that this approach identifies events that largely overlap with the ones studied in the rare disasters literature. Later we present macro risk estimates based on cutoffs of 1.5, 2.5, and 3 standard deviations, as well as based on the crises identified in Barro and Ursúa (2008). Our baseline specification relies on consumption series because asset pricing theory links the equity premium to aggregate consumption, but we also produce estimates based on GDP data. We also present estimates based on crises identified using standard deviations that remain fixed after WW2.
A common concern in the rare disaster literature is that macroeconomic crises are so rare that even pinning down the unconditional crisis probability poses a challenge. We gain traction by using a broad array of predictive variables. We posit a model in which the unobserved quantity of interest, $\pi_{i,t}$, is a function of the information available at year $t$. This information can be summarized by the vector $X_{i,t}$, i.e., $\pi_{i,t} = f(X_{i,t})$ for some function $f$. In our baseline specification, we postulate that $f$ takes the form of a standard normal distribution (i.e., a probit model). Eq. (3) therefore becomes:

$$\pi_{i,t} \equiv \Pr(\text{Crisis}_{i,t+1} = 1|X_{i,t}) = \Phi(X'_{i,t} \cdot b).$$

The fitted values in (3) yield our country-level risk estimates $\hat{\pi}_{i,t}$. (Throughout this paper, we use “hats” to denote the empirical counterparts of the true unobserved values.) The vector $X_{i,t}$ includes a large number of variables observable at time $t$, and is thus likely to encode more information than past realizations of consumption or output. Using a rich set of predictors also ensures that its span is as close as possible to the information set of economic agents $I_{i,t}$.

Overfitting is an apparent difficulty when working with a large number of predictors. To circumvent that difficulty, we exploit the panel structure of our data by assuming that the coefficient vector $b$ is the same for all countries. While homogeneity restrictions are unlikely to be literally true, in practice, they permit more efficient parameter estimation, at the cost of a hopefully small bias. Empirically, such restrictions often lead to improved forecasting performance (see, e.g., Garcia-Ferrer et al. (1987), for an early application to macro data).

B. Consumption and GDP Data

Our primary dataset consists of an updated version of the international panel on real per capita consumption expenditures and GDP that was constructed by Robert Barro and José Ursúa and described in Barro and Ursúa (2010). The sample covers 42 countries, for many of which data is available since the early nineteenth century. We extend the data set to the years 2010–2015—and thereby include the Great Recession—using the World Bank’s World Development Indicators and merging it with data on asset prices, wars, and other crises. (The Online Appendix provides details on data sources and the construction of all variables used in the analysis.) The data spans 1870 to 2015 and comprises 25 OECD countries, 14 countries from Latin America and Asia, as well as Egypt, Russia, and South Africa.

Our baseline crisis definition (2) identifies a large macroeconomic crisis as a decline in log consumption two standard deviation below a country’s long-term growth rate. (The Online Appendix reports long-term growth rates and consumption volatilities for all countries.) This
baseline definition yields 186 crises, which corresponds to a crisis frequency of 3.9%. On average, consumption falls by 14.7% during a crisis. These numbers are comparable to the 3.6% annual probability to enter a disaster and 21.9% average size reported in Barro and Ursúa (2008). Also, remark that the frequency of crises is larger than the corresponding percentile for the normal distribution (2.3%). This is due to leptokurtosis of consumption rather than skewness. We find that consumption growth, standardized using individual country moments, is slightly right-skewed with a skewness of 1.02, and highly leptokurtic with a kurtosis of 19.51.

Figure 2 gives an overview of macroeconomic crises over our sample. Small black dots indicate data unavailability, while blue dots represent crisis events (we report the list of crises in the Online Appendix). Data coverage begins at various times across countries but is generally continuous once it starts, except for Austria, Singapore, and Malaysia. We do not interpolate missing observations, although our results are unaffected when we do so. There is an apparent clustering of macroeconomic crises, in particular around the two world wars (highlighted by shaded areas), and crises are relatively less frequent in the post-1945 period. The Great Recession is associated with several international crises, although not in the United States. We find that the U.S. experienced three large macroeconomic crises, all within the prewar period. In the Online Appendix, we show that these crises coincide with the crises identified by Barro and Ursúa (2010). Section III.B presents macro risk estimates based on the crises identified in Barro and Ursúa (2008).

An open empirical question is to what extent international data is useful to quantify U.S. macroeconomic risk. The U.S. did not experience any major crisis (according to our definition) in the postwar period, but this is not statistically striking. For instance, if the probability of a crisis is 3%, the likelihood of a 75-year quiet spell is just above 10%. Regressing the crisis indicators on a United States dummy indicates that the U.S. realized crisis frequency (2.1%) is not statistically significantly different from the international frequency (3.9%). However, the average size of U.S. crises (7.8%) is significantly smaller, at the 5% level, than the international average (14.7%). We consider this difference when we calibrate our rare disaster model in Section IV.

C. Predictive Variables

We assemble a relatively large annual dataset of predictor variables spanning 1876 to 2015. Table I gives an overview of our sample. We select variables based on their availability of comparable data for all countries over the most extended sample available. We group them in five categories: Macro; War and Political; Financial Conditions; Natural Disaster; and Asset
Price. The Online Appendix provides further details.

Our sample of macro predictors aims at capturing serial and spatial correlation in macroeconomic risk. “Crisis” is the lagged dependent variable, which is an indicator equal to one if a crisis occurred in the previous year (see Eq. (2)). We find that 31 crisis years out of 186 were preceded by a crisis year. “Recession” is a variable equal to one when a country experienced a negative growth rate in the previous year. Recession events are quite frequent; Table I indicates that 28.70% of country-years are in a recession on average. “Crisis abroad” captures the possible transmission of the realization of a foreign crisis. It is constructed as a distance-weighted average of the crises that occurred abroad:

$$\text{Crisis abroad}_{i,t} = \sum_{j \neq i} d^{-1}_{j,t} \text{Crisis}_{j,t},$$

where $d_{j,t}$ measures the geographic distance between countries, normalized such that $\sum_{j \neq i} d^{-1}_{j,t} = 1$, $\forall j$. We also include predictors based on past consumption growth rates, and a “world” growth rate constructed by GDP-weighting consumption growth for the countries in the sample with available data.

We create three variables encoding wars, civil wars, and political crises, obtained primarily from Sarkees and Wayman (2010) and the Center for Systemic Peace (CSP). Our wars and civil wars data consist of armed conflicts that resulted in battle deaths. We define political crises as events of power disruption (e.g., a military coup) as well as periods of political instability (e.g., a country under occupation by a foreign power). We also construct a variable capturing foreign wars and political crises as in Eq. (4), where Crisis is replaced by a variable equal to one whenever a foreign country enters into one of the three above crisis. With the exception of coups and similar events, these data describe spells that typically last for several years. Though defined based on systematically recorded crisis events, they might inadvertently contain a hindsight bias, giving the econometrician information that is only available ex post. We thus set the corresponding crisis indicators to one for the duration of the crisis after the first year. We further show in Section III.B that our predictor set is rich enough that our estimates do not change much when removing variables that could be subject to hindsight bias.

Financial condition predictors include crisis dummies capturing banking, currency, and sovereign default crises, as well as extensive inflation periods. Our primary data source is Reinhart and Rogoff (2009), but we also checked financial crisis indicators obtained from the Jordà-Schularick-Taylor Macrohistory Database (Jordà et al., 2017). We also create a variable proxying for credit conditions. Recent evidence indicates that periods of accelerated credit
growth are more likely to be followed by slower growth, deeper recessions, and financial crises (e.g., Gourinchas and Obstfeld, 2012; Schularick and Taylor, 2012). We compute the three-year changes in bank loans-to-GDP ratio over three years using data from the Jordà-Schularick-Taylor database. The database covers 17 advanced economies, and we construct a GDP-weighed global factor, which we assign to the 42 countries in our sample.

We also create two variables capturing natural disasters. A “natural disaster” indicator equals one when a major earthquake, tsunami, or volcano eruptions occurs in a given country. We focus on disasters that reached the highest grade in event classifications from the National Centers for Environmental Information. We also set our “natural disaster” indicator to one for all countries during the Great Influenza Epidemic of 1919-1920. We construct an indicator variable capturing the most devastating famines that occurred over the sample. Similarly to war and political crises, we initiate natural disaster indicators after the first year, e.g., we start the Great Influenza indicator in 1919 although the pandemic began in 1918.

Our primary source for asset price data is Global Financial Data. To maximize sample size, we work with U.S. series to proxy for world variables. Estimates based on individual country predictors yield similar results, albeit on shorter periods. Asset pricing theory and empirical evidence suggest the dividend-price ratio is a standard proxy for the equity premium (e.g., van Binsbergen and Koijen, 2010). If the equity premium varies in proportion to disaster concerns, then the equity premium, and thus the dividend-price ratio, should forecast future crises. Our proxy for the world dividend-price ratio is the S&P 500 dividend-price ratio. We also construct a volatility index based on the CBOE VXO index from 1986. In the spirit of Bloom (2009), we extend the series back to 1871; we use daily data from 1885 through 1927, and monthly data before 1885, following Schwert (1989). Finally, we use a yield-curve slope predictor (e.g., Harvey, 1988; Estrella and Mishkin, 1998), based on the difference between the ten-year Treasury bond and the one-year short rate.

II. Forecasting Large Macroeconomic Crises

Our first goal is to assess whether macroeconomic crises are statistically predictable, and if so, which variables matter to forecast them. We first present in-sample estimates of the model exposed in the previous section. We next ask whether crisis forecastability carries over out of sample.

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5 We also considered a separate pandemic indicator variable featuring the global pandemics studied in Jordà et al. (2020), with unchanged results.

6 GFD offers a world dividend-price ratio, which is available later, from 1925. In our sample, the correlation between the two series is 0.91.
A. In-Sample Results

We report regression results of the panel probit forecasting model (3) on the panel of 42 countries over the period 1876-2015. The dependent variable is an indicator equal to one when there is a macroeconomic crisis in country \(i\) at time \(t+1\), and otherwise zero. Our panel consists of 4,695 observations of which 186 are realized crises. For the ease of interpretation, we directly report marginal effects at the means of predictive variables, rather than probit coefficients. Standard errors are clustered by country and year.

Figure 3 shows univariate specifications results for each predictor variable. To facilitate comparison between variables, we report standardized marginal effects, so that each dot represents the effect of a one standard deviation increase in the variable of interest on the likelihood of a macroeconomic crisis. The figure also shows 90% confidence bands. With one exception, all variables have the expected sign and forecast macroeconomic crises with somewhat similar magnitudes.\(^7\) For instance, the standardized effect of a prior crisis is 0.013. Given that the standard deviation of the crisis variable is 0.193 (see Table I), this means that being in a crisis raises the future probability of a crisis by \(0.013/0.193 \approx 6.9\%\), which is about twice the unconditional crisis likelihood. The coefficient is not only economically but also statistically significant \((t\text{-stat} = 5.2)\). Past recessions have an even greater forecasting power. We also find evidence of the transmission of foreign crises, as well as substantial forecasting power of past consumption growth rates. Among the remaining group of variables, War, War/political crises abroad, and the S&P 500 dividend-price ratio have the larger forecasting power. Being at war raises the likelihood of a crisis by about the same proportion as being in a macroeconomic crisis (6.4%). A one standard deviation increase in the U.S. D/P ratio raises the likelihood of a disaster by 2.1\% \((t\text{-stat} = 7.1)\). This result is consistent with rare disaster models, which posit a sharp link between the equity premium (proxied by the D/P ratio) and the likelihood of a disaster. Also, in line with rare disaster models (Barro and Liao, 2019; Farhi and Gabaix, 2016), we find that stock volatility and currency crises forecast future crises.

We now turn to our baseline predictive model, which we use to construct our \(\pi\)-estimates. In this context, we are mostly interested in spanning the information set of economic agents. We do so by means of a saturated multivariate specification using all (i.e., nineteen) predictive variables simultaneously. While, in a time-series context, such a specification would be likely to suffer from in-sample overfitting, the number of variables remain small relative to the number of countries (42) in our sample. In the next subsection, we nevertheless verify that this specification

\(^7\)The exception is Credit growth, which we expected to increase the likelihood of a crisis. In an unreported regression, we find that Credit growth significantly forecasts banking crises, in line with, e.g., Schularick and Taylor (2012).
keeps its ability to forecast crises out-of-sample.

We report standardized marginal effects in Table II. The first column reports estimates for the entire sample. Throughout this table, we continue to estimate the model in subsamples, including the periods before and after 1945 and separating OECD and non-OECD countries. We also consider alternative crisis definitions, including 1.5, 2.5, and 3 standard-deviation cutoffs and a specification with a 2-SD cutoff, but that assumes no break in consumption volatility around WW2. Finally, we show results for crises defined as 2-SD drops in GDP growth per capita. For each specification, we show the total number of observations, the number of countries, as well as the number of macroeconomic crises. The table reports the pseudo-$R^2$, but we note that $R$-squareds are poor candidates to assess the statistical fit for forecasting models in which the binary outcomes are unbalanced. We thus also report the volatility of the estimated macro risk (averaged across countries), which expresses the same information but presents the benefit of being more economically interpretable. The table also reports the correlation of the various risk estimates with the baseline model estimates. Last but not least, we report our preferred forecastability statistics, the area under the ROC curve (AUROC) metric. The Receiver Operating Characteristic (ROC) is a standard tool to assess the accuracy of a binary classification system. The AUROC metric has a value of 0.5 for a model that has no discriminatory power and values of 1 for a model with a perfect fit. (The Online Appendix provides further details.)

Table II indicates that that our predictive model does have explanatory power, with an AUROC statistics of 0.806. To get a sense of magnitudes, Schularick and Taylor (2012) find that credit growth helps predict financial crises with an already high in-sample AUROC of 0.717. This excellent forecasting ability extends to different subsamples (Columns 2-5), and alternative crises definitions (Columns 6-10), with AUROCs statistically distinct from 0.5. We find that most candidate predictors keep their statistical significance when estimated jointly. The various specifications produce estimates of macro risk that are quite variable, with volatility ranging between 4.1% and 7.4% with a 2-SD cutoff. Macroeconomic crises are about as forecastable in postwar data than in prewar data, although the postwar period features fewer crises, resulting in less volatile dynamics. Across specifications, risk estimates are very similar to the baseline model estimates (correlations above 0.88).

B. Out-of-Sample Results

So far, we have only considered in-sample regressions, which means our forecasts may be affected by look-ahead bias, even though we only use lagged data. Another concern, given the large
number of predictor variables in our model, is overfitting. In this subsection, we perform out-of-sample forecasting exercises to verify that our model provides accurate early-warning signals of upcoming crises.

We begin by forecasting over an expanding window to assess if our model could have been used in real-time. We start forecasting in 1922 rather than 1876, which gives us a 50-year training period (3,621 country-observations and 134 crises) to estimate our model. Each year \( t \), we estimate Eq. (3) using data available up to \( t \) and use coefficient estimates to forecast crises in \( t + 1 \). These forecasts constitute our out-of-sample estimates of the local probabilities \( \hat{\pi}_{i,t} \).

This first approach leaves open the objection that we keep using full-sample consumption moments to date crises—our dependent variable—so that our results may still suffer from look-ahead bias. The reason we do so is to maximize sample coverage. Identifying crises in real-time requires us to use a training period to compute consumption moments in each country, which costs us more observations. We nevertheless repeat the above exercise with a smaller sample where we require 50 years of data per country (3,245 country-observations and 65 crises). We estimate consumption growth and consumption volatility on an expanding window (we do not assume a change in volatility after 1945 as we do in our baseline model). Doing so ensures that the dependent variable is also constructed in real-time.

A limitation of the two out-of-sample tests outlined above is that they use the early part of the sample to train the forecasting model and test the forecast on the later portion. There may be differences in predictability between the early and late parts of the sample, which could be missed by these exercises. We thus perform a third test, based on cross-validation. For this test, we pick a random sample of 21 countries to train the model and test the model on the remaining 21 countries. We repeat this operation 250 times and report the averaged statistics.

We report the results of the three tests in Table III, which shows out-of-sample AUROC statistics, as well as in-sample statistics evaluated over the same sample. The forecasting performance is mechanically higher in-sample, but we observe that the model remains quite successful out-of-sample. For instance the in-sample AUROC equals 0.816 in the baseline specification, and declines to 0.775 out-of-sample. In the three configurations, we find that forecasts remain significant at the 1% level. Beyond statistical significance, these high scores imply that macroeconomic crises are predictable in real-time.

III. Macroeconomic Tail Risk over the Twentieth Century

In this section, we use the fitted values of our forecasting model to construct country-level macro risk estimates. We document new stylized facts about the behavior of macroeconomic tail risk
over the long run.

A. Macroeconomic Risk Estimates

Figure 4 displays the fitted one-year-ahead crisis probabilities (solid lines), together with realized crises (shaded areas) for the 42 countries in our sample. The crisis probabilities \( \hat{\pi} \) constitute our macro risk estimates and correspond to the fitted values in Eq. (3). (Figure 1, which we presented in the introduction zooms in on U.S. risk.) Our baseline specification utilizes the full range of predictor variables, and the left column in Table II shows point estimates.

Macroeconomic tail risk is volatile, countercyclical, and persistent. Crisis probabilities \( \hat{\pi} \) typically range between 0% and 10%, and occasionally spike higher during realized crises, reflecting the serial correlation present in the data. Regressing \( \hat{\pi} \) on time fixed effects yields a 56% R-square, meaning that about half of the variance in country risk is due to common forces. We obtain a similar number when we regress the U.S. estimates on the average cross-country probability (see the Online Appendix for a graphical comparison). This number is consistent with Lewis and Liu (2016), who show that a high degree of common macroeconomic risk is necessary to reproduce the fact that international asset return correlations are higher than consumption growth correlations.

B. Robustness

We noted in Section II that coefficient estimates are relatively stable when estimated in subsamples. The Online Appendix confirms that the corresponding macro risk estimates are comparable across subsamples. We further show that estimates excluding asset prices are almost indistinguishable from our main estimates. We believe this is important to assess the role of macroeconomic tail risk in asset pricing, as discussed in Section IV. The Online Appendix shows that series based on linear probability and logistic models, and series constructed from quantile regressions, following Adrian et al. (2019) are all very similar. In addition, the real-time estimates constructed in Section II.B exhibit comparable trajectories.

We also compare our baseline crisis definition in Eq. (2) to definitions based on cumulative drops in consumption that have been offered in the rare disasters literature. In Section I.B, we noted that our 2-SD crises are about equally rare as the ones identified in Barro and Ursúa (2008), which are defined as cumulative consumption declines exceeding 10%. In fact, for the period during which the two samples overlap, 69% of the one-year crises are also in Barro and Ursúa’s list of peak-to-trough macroeconomic crises. We next construct risk estimates using peak-to-trough macroeconomic crises. An immediate difficulty is that the peak-to-trough
methodology identifies crises ex-post and that crises typically last more than one year, whereas our approach identifies one-year crises in real-time. To make the comparison meaningful, we must decide at which point in time the economy enters a peak-to-trough crisis. We consider two indicators, one equal to one the year immediately after the peak and a second indicator equal to one the second year after the peak. The Online Appendix shows our baseline macro risk together with estimates constructed with each indicator. While both indicators suffer from look-ahead bias, we find that the first one is much more forecastable than the second. (For instance, the pseudo $R^2$ is 0.31 for crises identified one-year after the peak, against 0.17 for crises of the second type.) This translates into sawtooth patterns for the first series. The second series, in contrast, is quite similar to our baseline estimates, one exception being WW2, which appeared relatively safer according to Barro and Ursúa crises.

Another potential concern is a look-ahead bias among some of our predictor variables. While we construct our variables with this concern in mind, the possibility remains that some may still include information about the future, leading us to overestimate the degree of crisis predictability. In the Online Appendix, we report the results of two additional exercises. First, we drop all suspicious variables—wars and political crises, and natural disasters, as well as the financial conditions crisis indicator. This results in a risk estimate that is fairly close to our baseline, which indicates a moderate loss in forecasting power. (For the full sample, the AUROC equals 0.771, against 0.816 when we include all predictors.) Look-ahead bias is also a lesser concern if macroeconomic crises are forecastable over horizons exceeding one year. We thus construct the probabilities of macroeconomic crises two and three years ahead. Remarkably, these longer-horizon probabilities are quite volatile, which again indicates substantial forecasting power. The corresponding AUROCs equal 0.725 and 0.702 at the two- and three-year horizons and are both statistically significant at the 1% level. The probabilities are reasonably similar, the main difference being the Great Depression, which would have surprised a forecaster at a three-year horizon. Our overall conclusion is thus that look-ahead bias does not materially affect our results.

C. Comparison with Option-Based Estimates

Our estimate $\hat{\pi}$ can be interpreted as the rational expectation probability of a macroeconomic crisis, provided that our predictor set spans the information set of economic agents. Another natural approach to recover the rational expectation crisis probability is to use option prices. The price of derivatives, in particular deep out-of-the-money options, reveals the probability of a stock market crash, taken from the viewpoint of a risk-neutral agent, i.e., under the risk-
neutral measure. Under assumptions about investors’ preferences, it is possible to recover the physical probability of a stock market crash (Bollerslev and Todorov, 2011). Imposing further assumptions about the link between stock returns and consumption, one can compute the physical probability of a consumption crisis. Backus et al. (2011) tackles the challenge, but they assume a constant crisis probability. Siriwardane (2015) relaxes this assumption, but focuses on the risk-neutral crisis probability, while Barro and Liao (2020) back out the time-varying probability of a macroeconomic disaster, under the physical measure.8

Barro and Liao produce risk estimates for six countries over the 1994-2018 period, which we report in Figure 5 (black lines) next to ours (blue lines). Their model assumes that rare disasters obey a power-law distribution, as in Barro and Jin (2011). On average, option-implied probabilities equal 6.1%, which is higher than the unconditional probability in Barro and Jin (2011) (3.8%) and our mean crisis probability (3.9%). They are also more volatile, with several large spikes exceeding 20%. The presence of such spikes is in part due to the fact that option based estimates are sampled monthly, and in comparison, our yearly estimates appear more muted. We also observe that option-implied probabilities strongly co-move together (i.e., the R-square in a regression of option-implied probabilities on time fixed effects is 85% whereas it is 56% for macro risk).

The benefit of option-based estimates is that they do not require assumptions about investors’ information set. This benefit can turn into a weakness if option prices sometimes fluctuate for reasons beyond macro risk, which seems plausible. For instance, the option-based estimates indicate that macroeconomic risk was about equally high in the U.K., Germany, or Sweden during the LTCM crisis of 1998 as during the Great Financial Crisis. Likewise, using a longer U.S. sample that extends to 1983, Barro and Liao report that the probability of a disaster in the next month reached 89% at the time of the October 1987 stock-market crash.

Our approach instead takes option prices as one element in investors’ information set. We use a volatility index to proxy for option prices over the long-run. This index is based on stock realized volatility before 1986 and afterward on the VXO index from 1986 onwards. The VXO is based on the S&P 100, a narrower index than the S&P 500 used by Barro and Liao (2020), but otherwise very similar. The correlation between the VXO and the monthly U.S. option-implied disaster probability is 0.92 over our sample period (see the Online Appendix).9 This means the option-based estimate arises approximately as a special case in our predictive framework, in which the only useful predictor is the volatility index.

8Relatedly, Seo and Wachter (2019) show how assuming a time-varying disaster probability resolves several option market puzzles.
9Barro and Liao (2020) and Siriwardane (2015) report comparable correlation with the VIX, which is based on the S&P 500.
To see if this special case is true, we would ideally like to test if option-implied probabilities forecast future crises and if other variables have incremental forecasting power for future crises. However, we only observe a single crisis over the short sample in which options are available. A sensible alternative is to substitute the volatility index to option-implied probabilities, especially given that option prices are highly correlated across countries. The results, which appear in Table II, indicate that volatility is among the most robust predictor over our full sample.

To further test if other variables display incremental forecasting power, we construct one crisis probability that excludes the volatility index, and another based exclusively on the index. We next use the ROC methodology to compare the forecasting power of rules based on the two probabilities. We find that the former probability has about the same forecasting power as our baseline specification (AUROC = 0.806). Instead, the volatility index produces a statistically significant, but quite lower AUROC of 0.61. A Wald test rejects the null that the probability solely based on the volatility index has equal forecasting performance, meaning that option prices do not fully summarize macroeconomic tail risk.

D. Consumption and Output Growth

We next study the link between macro risk and the first moment of macroeconomic growth. We run predictive regressions of $H$-year ahead log consumption and output growth on past country crisis probabilities $\hat{\pi}_{i,t}$:

$$\sum_{h=1}^{H} \Delta \log c_{i,t+h} = a_i + b\hat{\pi}_{i,t} + u_{i,t+H}. \quad (5)$$

Table IV presents estimation results at 1-, 3-, and 5-year ahead cumulative horizons. For comparison purposes, we also report similar results for cumulative crises. This exercise thus allows us to see if crisis predictability carries over at horizons longer than one year. We report slope estimates, standard errors, and $R^2$ statistics for the full sample (1876–2015) as well as individually for the pre-war (1876–1945) and post-war (1945–2015) periods.

We see that an increase in macro risk predicts a decline in consumption growth, but the effect dies out over time, and magnitudes are relatively small. For instance, the full-sample coefficient for consumption growth is $-0.19$, meaning that a one standard deviation increase in the U.S. $\hat{\pi}$ (5.7%) lowers next-year growth rate by 1.1%. We find that macro risk holds its forecasting power over longer horizons for output growth, but with similar magnitudes. In contrast, macro risk has robust forecasting power for future crises in all specifications. The slopes are about one at the 1-year horizon (they don’t precisely equal one because crisis probabilities are estimated with a probit model) and increase monotonically with the forecasting horizon.
We can relate these results to the well-known disconnect between asset prices and macroeconomic outcomes. In particular, Beeler and Campbell (2012) emphasizes the limited ability of the U.S. dividend-price ratio to forecast future dividends and consumption, meaning that most of the variation in the D/P stems from changes in risk premia. We show in Section IV that time-varying disaster models can reproduce this feature of the data. Before turning to the model, we establish the link between macro risk and the equity premium in the remainder of this section.

E. Macroeconomic Risk and Equity Premium Proxies

Rare disasters model predict that the equity premium varies over time in tandem with macroeconomic tail risk. The reason is that investors shun stocks when the probability of a disaster increases, effectively raising the equity premium. In what follows, we ask if this link is present in the data. Since the equity premium is unobserved, we use three candidate proxies: the dividend-price ratio and two volatility indices. In a recent paper, Martin (2017) demonstrates a new link between market volatility and the equity premium. Specifically, the risk-neutral market return variance scaled by the gross risk-free rate constitutes a lower bound on the equity premium. The empirical counterpart to this bound is a volatility index, SVIX$^2$, that can be computed from option prices and is available on Martin’s webpage from 1996 to 2012. To obtain a longer time series, we rely on our volatility index squared, which we multiply by the real U.S. risk-free rate and scale to have the same mean and standard deviation as the SVIX$^2$. This longer series provides a reasonable approximation for the SVIX$^2$ (corr. = 0.82).

Figure 6 plots the probability of a macroeconomic crisis in the United States, together with the S&P 500 dividend-price ratio in Panel A, and with the two volatility indices (Panel B). Because we use both the D/P ratio and the volatility index to construct our baseline crisis probability, $\hat{\pi}$, we present in Figure 6 a variant excluding asset price predictors, denoted $\hat{\pi}^-$. We see that macro risk is positively related to both equity premium proxies. For instance, both macro risk and the D/P ratio spiked during the Great Depression and the two world wars. Symmetrically, the 1990s stock market boom (low D/P) coincided with a marked decline in macroeconomic risk during the Great Moderation, as previously noted by Lettau et al. (2008). The correlation coefficient between the two series is about 0.39, while the correlation with $\hat{\pi}$ (which includes asset price predictors) is 0.50. Likewise, we observe a positive albeit imperfect correlation between macro risk and the SVIX$^2$. Both rise during the Great Recession, but we observe relative disconnects during the 1997 Asian financial crisis and the European debt crisis, which do not appear as periods of elevated macro risk in the U.S. Further, squared volatility did
not increase during the two World War, in contrast with macro risk and the D/P ratio. This is in line with our earlier observation, in Section III.C, of a strong but imperfect association between our volatility index and macro risk. The SVIX index is also noticeably less persistent, with a half-life of about six months, as compared with a half-life of 3 years for the D/P ratio. Consequently, our baseline macro risk measure, which captures the information of the D/P ratio and volatility, exhibits a persistence that lies in between (half-life of 1.7 years).

F. Macroeconomic Risk and Stock Returns

We can further test the link between macroeconomic tail risk and the equity premium using stock returns. We first test if stock markets fall around macroeconomic crises, and then ask whether macro risk forecasts future stock returns. We obtain stock market and one-year interest rate data from Global Financial Data, which we use to construct excess stock returns series. We find total return and short rate data for 40 (out of 42) countries, corresponding to a smaller panel of 1,667 year-country observations. These are generally broad-market, market capitalization indices such as the S&P 500 and the FTSE All-Share index (for the U.K.). However, these data are less comprehensive than macro series and often contain gaps, in particular during wars.

We first investigate how excess stock returns behave around macroeconomic crises. In a recent paper, Barro and Ursúa (2017) find that major depressions indeed tend to coincide with stock market crashes. They note that macroeconomic crises are periods for which stock return data is most likely to be either missing or inaccurate, for example, due to market closure, which is why they measure depressions and crashes over multiple years. To complement their results, we adopt a flexible specification with five leads and lags around the year of a large macroeconomic crisis:

$$r_{i,t} - r_{i,t}^f = a_i + \sum_{j=-5}^{5} b_{-j} \text{Crisis}_{i,t+j} + u_{i,t},$$

where $r_{i,t} - r_{i,t}^f$ is the local currency excess return in country $i$ in year $t$. The $b$-coefficients in this regression, which we show in Figure 7, indicate how stock returns evolve around macroeconomic crises. We observe that stock prices tend to anticipate macroeconomic crises, with negative returns of 28.1% on average the year preceding a crisis. Figure 7 also indicates that excess stock returns rebound right after the typical crisis. The presence of a stock market crash followed by a rebound supports the idea that the equity premium increases when the economy enters a crisis.

We explore this hypothesis further in Table V, where we forecast stock returns in excess of the risk-free rate. The table has a similar layout as Table IV. We estimate predictive regressions
of cumulative log excess returns on lagged macro risk over horizons of 1, 3, and 5 years. The top panel uses our baseline $\hat{\pi}$ probability, while the second panel uses the measure excluding asset price predictors, $\hat{\pi}^-$, which we introduced in the previous subsection. In the next two panels, we separate the effect of macro risk from realized crises.

We see that the macro risk strongly predicts future returns. A one percent increase in $\hat{\pi}$ forecasts a 0.86 percentage point larger return the next year, the forecasts adding up to 3.21 percentage over a five-year period. The regression coefficient suggests a volatile equity premium. For instance, given the volatility of U.S. $\pi$-estimates (5.7%), the 0.86 coefficient implies a volatility for the one-year equity premium of about 4.8%. This is close to results in the prior literature. For example, using the U.S. dividend-price ratio, Cochrane (2011) estimates that the U.S. equity premium varies by about 5.4% per year. One concern could be that $\hat{\pi}$ forecasts returns only incidentally because it is also correlated with asset prices, which themselves contain information about future stock returns. This concern is only valid to the extent that our set of predictor variables is much smaller than the information set of market participants, in that asset prices carry too large a weight in $\hat{\pi}$ than in the true crisis probability. We address this concern by forecasting returns with $\hat{\pi}^-$. Our conclusions remain mostly unchanged, with only slightly smaller coefficients estimates. We see also in Table V that predictive coefficients are higher in the postwar period. One difference between the two subperiods is that much fewer disasters occurred during the latter (Table II). As we shall see in the next section, rare disaster models can qualitatively reproduce this feature of the data, with higher predictive coefficients in the absence of disasters.

We next ask whether return predictability reflects actual macroeconomic tail risk or the realization of macroeconomic crises. Rare disaster models predict that the equity premium varies over time because macroeconomic tail risk changes over time. Another plausible channel is that agents sell risky assets during recessions, causing asset prices to fall. In other words, agents require a higher risk premium during crises, not because of the risk that their economic situation worsens, but because their willingness to take risk has decreased. Models of habit formation such as Campbell and Cochrane (1999), for instance, predict that, during recessions, agents’ consumption approaches their habit level, effectively raising their risk aversion. In this class of models, movements in risk aversion rather than movements in risk cause the equity premium to move over time. This prediction is consistent with the pattern in Figure 7 that stock prices fall and rebound around crises. We investigate this mechanism with the third specification in Table V, where we forecast excess returns with an indicator variable corresponding to lagged realized crises. The associated coefficient is large and statistically significant. Expected returns
are 18.4% higher, a year after a macroeconomic crisis. However, this effect mostly disappears once we control for $\hat{\pi}$, in particular in postwar data. This suggests that agents primarily care about macroeconomic tail risk rather than realized crises, as predicted by rare disasters models.

**IV. Asset Pricing Implications**

In this section, we evaluate the asset-pricing implications of a workhorse equilibrium asset pricing model with time-varying macroeconomic tail risk. The model is a discrete-time version of the one offered by Wachter (2013) and belongs to the discrete-time affine class proposed in Drechsler and Yaron (2011). After describing the economy and the dynamics of aggregate consumption, we derive equilibrium asset prices. In the spirit of Mehra and Prescott (1985), we next ask if a calibrated economy can reproduce key asset-pricing moments. To do so, we take advantage of our methodology, which can produce macro risk estimates that do not exploit asset price information.

**A. Model**

We consider a pure exchange closed economy, à la Lucas (1978), in which log consumption growth, $\Delta c_{t+1}$ evolves as follows:

$$\Delta c_{t+1} = \mu + \sigma \varepsilon_{t+1} + v_{t+1}, \quad (7)$$

where $\varepsilon_{t+1}$ and $v_{t+1}$ are two mutually independent shocks. The first shock is a standard normal random variable, and the second shock captures rare consumption disasters. We model $v_{t+1}$ as a compound Poisson shock: $v_{t+1} = J_{t+1} \mathbf{1}_{\Delta n_{t+1} > 0}$, where $n_{t+1}$ is a Poisson counting process such that $\Delta n_{t+1} > 0$ describes a disaster event occurring at time $t+1$. The probability that the economy encounters a disaster in $t+1$, $\pi_t$, follows a discretized square-root process:

$$\pi_{t+1} - \bar{\pi} = \rho(\pi_t - \bar{\pi}) + \nu \sqrt{\pi_t} u_{t+1}, \quad (8)$$

where $\bar{\pi} > 0$, $0 < \rho < 1$, and $\nu > 0$ are constants and where $u_t$ is a standard normal random variable uncorrelated with $\varepsilon_t$ and $v_t$.

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10 We commit a slight abuse of notation since $\pi_t$ is only an approximation of the conditional probability of a disaster on the unit time interval (i.e., yearly). The Poisson counting process $n_{t+1}$ has intensity $\pi_t$. The exact conditional probability of a single disaster occurring over the horizon $\tau$ is therefore $\pi_t \tau \exp(-\pi_t \tau)$, while the probability that a disaster does not occur is $\exp(-\pi_t \tau)$. Hence the residual probability that more than a single disaster occurs is $1 - \exp(-\pi_t \tau)(1 + \pi_t \tau)$. According to our estimates, the latter value is about 0.05% when $\pi_t$ is at its steady state and less than 1% when $\pi_t$ is at its 99th percentile. So to facilitate terminology and ease the notation, we shall use $\pi_t$ for the conditional disaster probability and $\sum_t \Delta n_{t+1}$ (instead of $\sum_t \mathbf{1}_{\Delta n_{t+1} > 0}$) for the number of disasters.
Finally, disaster size $J_{t+1}$ follows a shifted negative gamma distribution with moment-generating function given by

$$\varphi(u) = e^{-u\theta}(1 + u\beta)^{-\alpha},$$

(9)

where disasters have support on $(-\infty, -\theta)$ and where the mean and variance are equal to $-(\theta + \alpha\beta)$ and $\alpha\beta^2$, respectively.

These dynamics are fairly standard and can potentially capture several asset pricing regularities (Gabaix, 2012; Wachter, 2013). Our model is most closely related to the continuous-time model of Wachter (2013). Gabaix (2012) calibrates a richer model that allows for movements in the disaster probability and the expected disaster size. The literature contains several alternative specifications. For example, Barro and Jin (2011) consider different laws for disaster size, such as single and double power laws. Gabaix (2011) and Gourio (2012) introduce time-varying disaster risk in production economies. Gourio (2008), Nakamura et al. (2013), Branger et al. (2016), and Hasler and Marfè (2016) consider more complex disaster dynamics, including unfolding consumption declines and subsequent recovery, as well as consumption declines leading to economic regime changes. We intentionally keep the model simple, tractable, and parsimonious to focus on the time-series relationship between disaster probability and equilibrium asset prices.

We assume the economy is populated by a representative investor with recursive preferences (Epstein and Zin, 1989):

$$V_t = \left[ (1 - \delta)C_t^{1-1/\psi} + \delta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}} \right]^{1/(1-1/\psi)}.$$  

(10)

In this expression, $\delta, \gamma \neq 1$, and $\psi$ respectively capture time discounting, relative risk aversion, and the elasticity of intertemporal substitution. To ensure tractability, we focus on the case of $\psi$ equal to one. Similarly to Collin-Dufresne et al. (2016), we normalize utility $V$ by consumption level $C$ such that the log value function $vc_t \equiv \log V_t/C_t$ is given by

$$vc_t = -\frac{\delta}{1-\gamma} \log \mathbb{E}_t[e^{(1-\gamma)(\Delta c_{t+1} + vc_{t+1})}].$$

(11)

The aggregate dividend paid by the equity claim is given by

$$\Delta d_{t+1} = \phi \Delta c_{t+1},$$

(12)

as in Abel (1999), Campbell (2003), Wachter (2013) and others. When $\phi > 1$, this ensures, in a parsimonious way, that dividends fall more than consumption when a disaster hits, as observed
in U.S. data (Longstaff and Piazzesi, 2004).

We solve for asset prices by expressing the stochastic discount factor in terms of the investor’s value function, which is affine in the disaster probability $\pi_t$. We next derive the return on the equity claim via the investor’s Euler equation up to the usual Campbell and Shiller (1988a) log linearization. The stochastic discount factor is given by

$$M_{t+1} = \delta e^{-\gamma \Delta c_{t+1}} \times \frac{e^{(1-\gamma)v_{c_{t+1}}}}{E_t[e^{(1-\gamma)(\Delta c_{t+1}+v_{c_{t+1}})}]}$$

(Collin-Dufresne et al., 2016) and the value function satisfies

$$v_{c_t} = v_0 + v_{\pi} \pi_t,$$

where

$$v_0 = \frac{\delta}{1-\delta} \left( \mu + \pi (1-\rho) v_{\pi} - \frac{\gamma \sigma^2}{2} \right),$$

$$v_{\pi} = \frac{\delta \rho - 1 - \sqrt{1 + \delta (2\beta \nu^2 + \rho^2 - 2\rho - 2\beta \nu^2 \gamma (1-\gamma))}}{\delta (\gamma - 1) \nu^2}.$$

Note that the stochastic discount factor variance (i.e., the priced risk in the economy) is state-dependent and increases with the disaster probability.

The log risk-free rate, $r_{f,t} = -\log E_t[M_{t+1}]$, is affine in the disaster probability:

$$r_{f,t} = -\log \delta + \mu - \gamma \sigma^2 + \pi_t (\varphi (1-\gamma) - \varphi (-\gamma)).$$

This risk-free rate is stationary and decreases linearly with the disaster probability $\pi_t$. An increased likelihood of a disaster increases the consumption risk, which the investor can hedge with risk-free investments. Consumption risk increases with the likelihood of a disaster, and so does the demand for safe assets increases, causing the risk-free rate to fall. This effect strengthens with the relative risk aversion coefficient.

To solve for the equity claim price, we log-linearize returns around the unconditional mean of the log D/P ratio $dp \equiv E[d_t - p_t]$ with $d_t - p_t \equiv \log D_t/P_t$:

$$r_{d,t+1} = \log(e^{-d_{t+1}+p_{t+1}} + 1) + d_t - p_t + \Delta d_{t+1}$$

$$\approx k_0 - k_1 (d_{t+1} - p_{t+1}) + d_t - p_t + \Delta d_{t+1};$$

$^{11}$The Online Appendix provides solution details.
where \( k_0 \) and \( k_1 \) are endogenous constants. Campbell et al. (1997) and Bansal et al. (2012) document the high accuracy of such a log linearization, which we assume to be exact. We use the Euler equation \( 1 = E_t[M_{t+1}e^{r_{d,t+1}}] \) to recover that the log D/P ratio is affine in the disaster probability:

\[
d_t - p_t = A_0 + A_\pi \pi_t,
\]

where

\[
A_0 = \log(1 - k_1) - \log(k_1) - A_\pi \bar{\pi} \quad \text{and} \quad A_\pi = \frac{1}{k_1^2 \nu^2} \left( \sqrt{\Omega^2 + 2k_1^2 \nu^2 (\varphi(1 - \gamma) - \varphi(\phi - \gamma))} - \Omega \right)
\]

with \( \Omega = 1 - k_1(\rho + (1 - \gamma)v_\pi \nu^2) \). The D/P ratio is stationary, and increases with the disaster probability \( \pi_t \) when \( \phi > 1 \). These dynamics reflect a preference for the early resolution of uncertainty about time variation in \( \pi_t \). An increase in disaster probability makes it more likely that crises will affect future consumption. An investor who prefers early resolution of uncertainty (\( \gamma > 1 \)) is worried about current disaster risk and uncertainty in future disaster risk. Hence equity prices are low relative to dividends when \( \pi_t \) is high (and vice versa). Remark that the substitution effect and the income effect offset each other because the elasticity of intertemporal substitution is equal to one.

The log equity premium is given by

\[
\log E_t[e^{r_{d,t+1}}] - r_{f,t} = \underbrace{\gamma \phi \sigma^2}_{\text{non-disaster risk}} + \underbrace{(\gamma - 1)k_1 A_\pi \nu^2 \pi_t \varphi^2}_{\text{disaster probability risk}} + \underbrace{(\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi - \gamma) - 1)\pi_t}_{\text{disaster size risk}},
\]

and the return variance is

\[
\text{var}(r_{d,t+1}) = \underbrace{\phi^2 \sigma^2}_{\text{non-disaster risk}} + \underbrace{k_1^2 A_\pi^2 \nu^2 \pi_t}_{\text{disaster probability risk}} + \underbrace{\phi^2 \pi_t \frac{\partial^2 \varphi(u)}{\partial u^2} \bigg|_{u=0}}_{\text{disaster size risk}}.
\]

Both the equity premium and the return variance are given by three terms. The first terms concern non-disaster risk and give rise to the usual Consumption-CAPM compensation (Lucas, 1978). The second terms reflect the compensation for the variation in disaster probability. Both terms increase with current disaster probability and also with its persistence and volatility for \( \gamma > 1 \). The third term is associated with disaster size risk and increases with both disaster size variance and the current level of disaster probability. Finally, the risk-neutral return variance
has a similar form:

$$\text{var}^t_q(r_{d,t+1}) = \phi^2 \sigma^2 + k_t^2 A_n^2 \nu^2 \pi_t + \phi^2 \pi_t \frac{\nu^2}{\mu} \varphi(u)_{u=\gamma},$$

(19)

where only the third term differs from the physical return variance. Hence, the variance risk premium is also proportional to $\pi_t$.

B. Calibration

Our approach to calibrate the model broadly follows Wachter (2013), with the key exception that we do not use asset prices to calibrate consumption dynamics, which we instead take from the data. The first group of parameters, $\mu$ and $\sigma$, determine consumption in normal times. We estimate them with maximum likelihood, using U.S. postwar consumption data, winsorized at the 2.5% level in both tails. We recover disaster risk parameters, $\bar{\pi}$, $\rho$, and $\nu$, using the macro risk estimates that do not use asset price information, $\hat{\pi}^-$. We back out the empirical counterpart of Eq. (8) by running the panel regression:

$$\hat{\pi}^-_{i,t+1} = -\rho \hat{\pi}^-_{i,t} + \rho \hat{\pi}^-_{i,t} + \nu u_{i,t+1},$$

(20)

Finally, we estimate the disaster size parameters, $\theta$, $\alpha$, and $\beta$, with maximum likelihood using the sample of realized crises.

Table VI presents the point estimates, standard errors, and 95% confidence intervals for the consumption process parameters defined by Eqs. (7)–(9). Consumption growth parameters in “normal times” belong to the usual range of values: the long-term consumption growth rate is 2.1%, and consumption volatility is 1.7%. The subsequent rows show that the key parameter estimates associated with disaster risk dynamics are close to the ones that Wachter (2013), and subsequent literature chose to match asset pricing moments. Disaster risk is both persistent ($\rho = 0.66$) and volatile, with a scale parameter $\nu$ of about 0.18. These two parameters contribute to the unconditional volatility of disaster risk, which is $\sqrt{\nu^2 \bar{\pi}/(1-\rho^2)} = 4.6\%$. The mean disaster size is 14.7% with a 9.8% standard deviation (the minimum disaster size is $\theta = 1.7\%$).

We plot the distribution of consumption declines, along with the fitted density, in the Online Appendix.

We complete the calibration by setting the leverage parameter $\phi$, and the preference pa-

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12Wachter chooses a more persistent $\rho = 0.92$ (using our notation), which is set to match the autocorrelation of the price-dividend ratio. The scale parameter is lower than in our setup ($\nu = 0.067$). Altogether, these parameters imply a somewhat smaller volatility of disaster risk (3.2%).
rameters $\delta$ and $\gamma$. Evidence from the Depression-era suggests that macroeconomic crises have a much larger effect on dividends than on consumption (Longstaff and Piazzesi, 2004). We follow Wachter (2013), who argues that a value of $\phi = 2.6$ is conservative and captures well the increased risk of dividends relative to consumption in both normal times and disaster periods.\(^{13}\)

Lastly, we calibrate the parameters $\delta$ and $\gamma$ to fit the main asset pricing moments. We find that reasonable values of relative risk aversion suffice to generate a high equity premium that conforms with the data. Hereafter we consider the pair ($\delta = 99\%$, $\gamma = 6$) as our baseline calibration.

### C. Asset Pricing Results

Table VII shows that this calibration leads to a good fit of the main unconditional moments of asset prices. This outcome is not surprising since our calibration happens to be relatively close to those used by Wachter (2013). Panel A reports historical asset pricing moments and the distribution of their model counterparts. In particular, we report several percentiles from 1,000 simulated paths of the economy of length equal to 1000 years. (We choose 1000 years to approximate the statistical power of the panel regressions in Section II.) To avoid generating a negative probability of disaster, we simulate monthly series and then convert them to annual frequency.\(^{14}\) In our baseline calibration, the median risk-free rate is about 0.4% with 2.5% median volatility. The median equity premium is 5.7%, and the median return volatility is 13.4%. The median D/P ratio is 2.1%, with 0.3% median volatility. The median first-order autocorrelation of the D/P ratio is 65%. The lack of persistence in comparison with the empirical data is because the D/P ratio is driven only by the disaster probability, which is less persistent than the observed D/P ratio (79%) in the data. In turn, the lack of persistence in disaster probability is also responsible for the gap in return volatility relative to the actual data. While our estimates of macroeconomic tail risk are strongly linked to the equity premium, we find that they leave room to additional mechanisms concerning a residual fraction of return volatility.

Variation in macroeconomic tail risk contributes to generating a high equity premium. Agents who prefer the early resolution of uncertainty not only dislike bad news, but also the possibility of more bad news in the future. They thus require compensation for the uncertainty associated with variation in the probability of a crisis. In the baseline calibration, we find that this uncertainty represents 31% of the steady-state level of the equity premium (according to

---

\(^{13}\)The correlation between U.S. dividend growth and consumption is 0.45 in our data; the model extension in Section IV.E relaxes the unit correlation assumption of the baseline model.

\(^{14}\)Simulations of $\pi_t$ at monthly frequency (i.e., $\pi_{t+\Delta} = (1 - \rho^\Delta)\bar{\pi} + \rho^\Delta \pi_t + \nu\sqrt{\pi_t}Z\Delta u_t$ for $\Delta = 1/12$) have a negligible likelihood (less than 0.1%) of realizations lower than zero, which we replace by a small positive threshold.
Eq. (17)) and 52% of the return variance at the steady-state (Eq. (18)). The compensation for actual disaster risk is in similar magnitudes, representing 62% of the equity premium level and 39% of the return variance. Non-disaster risk is qualitatively negligible in comparison, with respective shares of 7% and 9%.

To further understand the role of risk in the equity premium, we report in Panel B of Table VII the same moments, in the absence of realized disasters. A common peso argument is that average stock returns may be large by chance alone, in economies with few crises. The postwar U.S. economy is a case in point. We find that the equity premium is indeed slightly higher (the median is about 7.2%, against 5.7% in the baseline case in Panel A). Return volatility is also somewhat lower (the median is now 10.1% against 13.4% in Panel A), while other moments are essentially unchanged.

We next investigate the extent of predictability implied by the model. We showed in Tables IV and V that $\pi_t$ is a good predictor of excess returns but not of consumption growth. We run similar regressions of either cumulative excess returns or cumulative consumption growth rates, using horizons between 1 and 5 years. Figure 8 reports the 5th, 50th, and 95th percentiles of the predictive slope coefficients from the model simulations, together with their empirical counterparts, reproduced from Tables IV and V. In the left panel, we observe that disaster probability is a reliable predictor of excess returns. The distribution of the slope coefficients has support on positive values across all the horizons, meaning that the model reproduce the equity return predictability we observe in the data. Indeed, the point estimates from Table V feature a very similar pattern and belong to the interval produced by the model simulations at each horizon. In the right panel of Figure 8, we observe weaker evidence of consumption growth predictability, as in the data. Point estimates for the disaster probability are negative but close to zero, which is not surprising given the consumption dynamics of Eq. (7). The empirical counterparts from Table IV exhibit a similar pattern.

Figure 8 also plots the median predictive slopes from model simulations when realized disasters are turned off. In this case, consumption growth predictability coefficients collapse to zero; indeed, disaster probability cannot predict consumption growth at all in the absence of realized disasters. In contrast, excess return slope coefficients are slightly larger in the absence of realized disasters and thus compare well with the postwar data coefficients (Table V).

As noted in Section I, the average size of realized crises in the U.S. is usually smaller than in the entire panel. Thus, for the sake of robustness, we consider an alternative calibration where using more conservative estimates of disaster size that match the U.S. experience. Namely, we keep all parameters from our baseline setting (see Table VI) with the exception of $\alpha = 0.842$. 
Then, we also keep $\delta = 99\%$ and raise the relative risk aversion to $\gamma = 9$, which remains below the upper bound of 10 argued as reasonable by Mehra and Prescott (1985). The Online Appendix provides historical asset pricing moments and the distribution of their model counterparts. We observe that the model produces an overall good fit and still matches the levels of the risk-free rate (1.0%) and the equity premium (4.8%). Besides, predictability results are essentially unchanged.

D. Model Extension: the CRRA Case

A number of seminal papers, including Mehra and Prescott (1985), Barro (2006) and Gabaix (2012) consider the special case of constant relative risk aversion (CRRA). CRRA preferences arise when the representative agent’s elasticity of intertemporal substitution equals the inverse of the relative risk aversion coefficient ($\psi = 1/\gamma$). This restriction implies that a single parameter governs the representative agent’s attitudes towards risk and towards intertemporal substitution. In turn, the agent is indifferent regarding the timing of uncertainty resolution. The Online Appendix reports the full model derivation.

The main implication of introducing CRRA preferences is that the disaster probability $\pi$ is not anymore a priced state-variable; that is, the stochastic discount factor is not driven by the shock driving the dynamics of $\pi$ (i.e., $v_\pi = 0$). Nevertheless, both the log D/P ratio and the log equity premium remain affine functions of $\pi$. The log equity premium moves with $\pi$ only because changes in $\pi$ induce changes in consumption risk, which increases with $\pi$. However, the representative agent no longer fear the variation of $\pi$. The log equity premium becomes:

$$\log \mathbb{E}_t[e^{r_{d,t+1}}] - r_{f,t} = \frac{\gamma \phi \sigma^2}{\text{non-disaster risk}} + \frac{(\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi - \gamma) - 1)\pi_t}{\text{disaster size risk}},$$

which corresponds to Eq. (17) with $v_\pi = 0$.

Panel A of Table VIII provides historical asset pricing moments and the distribution of their model counterparts. We keep all parameter values from the previous analysis of the baseline model. The model produces a risk-free rate (about 4.5%) and an equity premium (approximately 4.4%) that are respectively above and below their empirical counterparts. We find that changing the relative risk aversion can improve the fit of one moment, but only at the cost of deteriorating the fit of the other. In addition, the risk-free rate becomes about twice more volatile than in the data.


E. Model Extension: the Cointegration Case

In this section, we keep preferences fixed, but relax the assumption (12) of a unit correlation between consumption and dividends. We assume instead that consumption and dividends are cointegrated. We do so for several reasons. First, recent evidence suggests that dividend growth risk decreases with the horizon (Belo et al., 2015; Marfè, 2017). In the baseline model, assumption (12) implies that dividend growth risk is higher than consumption growth risk at any horizon. Instead, cointegration preserves the relatively larger dividend risk at short horizons but imposes that consumption and dividend growth risk are equal in the long-run. This produces a downward-sloping dividend growth risk, in accord with the empirical evidence. Further, as documented by Gourio (2008) and Nakamura et al. (2013), disasters are often followed by recoveries. Cointegration generates dividend recoveries, as the dividend/consumption ratio increases after disasters, in line with the evidence in Longstaff and Piazzesi (2004) and Hasler and Marfè (2016).

In addition, relaxing the unit correlation between consumption and dividend growth lets us benchmark our model against an additional moment. In the previous section, we have highlighted the relatively high correlation between macroeconomic tail risk and the dividend-price ratio. For the U.S., the correlation with $\hat{\pi}$ is about 39%. Dynamics in our baseline model are summarized by a single state variable, $\pi$, which mechanically produces a unit correlation between $\pi$ and the dividend-price ratio. This assumption could make us overestimate the correlation between prices and discount rates and, hence, the effect of disaster risk on the equity premium. Introducing cointegration gives rise to an additional state variable, the dividend share of consumption, which breaks the perfect correlation between $\pi$ and the dividend-price ratio.

The model works as follows. Equations (7) to (10) continue to hold. Thus, the stochastic discount factor is unchanged and Eq. (13) to (15) remain valid. The new assumption concerns dividend dynamics:

$$\Delta d_t = \Delta c_t + \Delta s_t, \quad (22)$$

$$s_t = (1 - \phi)s + \phi s_{t-1} + \eta z_t + \tilde{v}_t, \quad (23)$$

where $z_t$ is a normal random variable and $s_t = \log D_t/C_t$ is the log dividend share of consumption, the second state variable, which captures mean-reversion of expected dividend growth. The presence of the shock

$$\tilde{v}_t = \kappa v_t = \kappa J_1 \Delta n_t > 0, \quad (24)$$

implies that dividends load more on disasters than consumption, for $\kappa > 0$. 


In equilibrium, the log D/P ratio is an affine function of both $\pi_t$ and $s_t$:

$$d_t - p_t = A_0 + A_\pi \pi_t + A_s s_t.$$  

(25)

In contrast, the log equity premium only depends on $\pi_t$:

$$\log E_t [e^{r_{d,t+1}}] - r_{f,t} = \underbrace{\gamma \sigma^2}_{\text{non-disaster risk}} + \underbrace{(\gamma - 1)k_1 A_\pi v \nu^2 \pi_t}_{\text{disaster probability risk}} + \underbrace{(\varphi(1 + \kappa(1 - k_1 A_s)) + \varphi(-\gamma) - \varphi(1 + \kappa(1 - k_1 A_s) - \gamma) - 1)\pi_t}_{\text{disaster size risk}}.$$  

(26)

This is because we make the simplifying assumption that the second and higher moments of dividend growth do not depend on $s_t$. The Online Appendix gives the full model solution.

Panel B of Table VIII reports historical asset pricing moments and the distribution of their model counterparts. We keep the parameter values for consumption and disaster probability dynamics, as well as preferences from the previous analysis of the baseline model. The additional parameters concern the dynamics of the dividend share of consumption. We set $\bar{s} = \log(0.10)$, $\phi = 0.97$, $\eta = 0.15$, and $\kappa = 2.6$ in order to capture the high persistence of the log D/P ratio and the correlation between disaster probability and the log D/P ratio.

We observe that the model produces an overall good fit for the main return moments. In particular, the model predicts a low risk-free rate and a high equity premium for a realistic coefficient of relative risk aversion. Under the baseline setting with $\gamma = 6$, the equity premium is about 5.2%. The cointegration assumption avoids overestimating the long-run dividend growth risk but implies a realistic equity return volatility.

Table VIII also indicates that the correlation between the log dividend-price ratio and the disaster probability $\pi$ compares well with its empirical counterpart (about 37.4% vs 39%). The cointegration model relaxes the unit correlation between the log D/P ratio and $\pi$ because the former is an affine function of both $\pi$ and the log dividend share of consumption $s$ (Eq. (25)). The correlation, therefore, depends on the log D/P ratio sensitivities with respect to each state variable. Of the two sensitivities, only the one with respect to $\pi$ depends on $\gamma$, because $\pi$ is a priced state variable. As a result, the correlation is positive and increasing with risk aversion, and $\gamma = 6$ reproduces the correlation we observe in the data.

F. Summary of the Calibration Analysis

Overall, our model supports the idea that macroeconomic tail risk is a key driver of the equity premium. In particular, the baseline calibration indicates the model can generate a large and
volatile equity premium. This does not come as a surprise: our calibration is close to Wachter (2013), which model can indeed rationalize some asset pricing puzzles. We contribute by using actual estimates of macro risk dynamics to calibrate our model, in the spirit of Mehra and Prescott (1985), and thus provide empirical support for the calibration of Wachter (2013) and subsequent literature.

Other aspects of the model are, of course, overly simplistic. In our baseline model, macroeconomic tail risk \( \pi_t \) is the only state variable, which means that all equilibrium quantities are perfectly correlated. We have emphasized in Section III.E that macroeconomic tail risk and the dividend-price ratio are positively, but imperfectly, correlated. The extended model we introduced in Section IV.E can reproduce this feature of the data. Section III.C also showed that many variables beyond option prices have forecasting power for future crises, which we interpreted as evidence of an imperfect correlation between macroeconomic tail risk and equity volatility. While the extended model still produces a unit correlation with volatility, a richer model, for instance assuming conditional heteroskedasticity in the dynamics of \( s_t \), could break this perfect correlation.

V. Conclusion

In this paper we propose a straightforward approach to measure macroeconomic tail risk over the long run (1876-2015). This approach is motivated by rare disasters models, but we have strived to ensure that our risk estimate is as comprehensive as possible and free from the restrictions of theoretical models. Our estimate captures the predictable variation in macroeconomic tail events, making use and summarizing the information content of a rich set of predictive variables.

We find that macroeconomic crises are comfortably predictable by variables such as lagged macroeconomic crises at home and abroad, wars and related disasters, and asset prices. By combining the information in our predictive variables, we obtain country crisis probabilities that capture the rich economic history of the 42 countries of our sample. Reflecting the predictive ability of its components, the crisis probabilities increase in periods of economic and political distress, and covary with asset prices, as implied by rare disasters models.

We then calibrate a rare disaster model using our estimates of macro risk dynamics. Doing so, we are careful not to use asset price information to ensure our calibration is not reverse-engineered to fit asset pricing data. Our model generates a high and volatile equity premium for a representative agent whose coefficient of relative risk aversion is 6 and whose elasticity of intertemporal substitution is 1. These results legitimate prior calibrations of rare disasters model that were set to match asset pricing moments.
Our approach is inspired by the long tradition in finance to estimate the equity premium using predictive regressions. Similar forecasting models are frequent in the context of recession and other crises forecasting. One innovation in this paper consists of showing that this type of model can measure economic “dark matter,” such as disaster risk. We leave to further research the study of other similar objects such as long-run risk (Bansal and Yaron, 2004), which our framework can accommodate as well. Finally, we note that rare disaster models can rationalize other regularities observed in the markets for bonds, options, and currencies (Gabaix, 2012; Gourio, 2013; Farhi and Gabaix, 2016), and offer the possibility of connecting macroeconomic aggregates with asset prices in production economies (Gabaix, 2011; Gourio, 2012; Kilic and Wachter, 2018; Isoré and Szczerbowicz, 2017). An empirical evaluation of these models is a fruitful area for further research.
Table I: Predictive Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Macroeconomic Predictors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crisis</td>
<td>Macroeconomic crisis</td>
<td>0.039</td>
<td>0.193</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Recession</td>
<td>Consumption growth rate &lt; 0</td>
<td>0.287</td>
<td>0.452</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Crisis abroad</td>
<td>Distance-weighted average of foreign crises</td>
<td>0.033</td>
<td>0.062</td>
<td>0.000</td>
<td>0.523</td>
</tr>
<tr>
<td>Consumption</td>
<td>Consumption growth</td>
<td>0.019</td>
<td>0.064</td>
<td>−0.557</td>
<td>0.487</td>
</tr>
<tr>
<td>Consumption (world)</td>
<td>GDP-weighted consumption growth</td>
<td>0.017</td>
<td>0.040</td>
<td>−0.213</td>
<td>0.116</td>
</tr>
<tr>
<td><strong>War and Political Predictors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>War</td>
<td>Interstate war</td>
<td>0.040</td>
<td>0.197</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Civil war</td>
<td>Intra-state or non-state war</td>
<td>0.059</td>
<td>0.235</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Political crisis</td>
<td>Political crisis</td>
<td>0.014</td>
<td>0.118</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>War/political crisis abroad</td>
<td>Distance-weighted average of foreign war/political crises</td>
<td>0.088</td>
<td>0.111</td>
<td>0.000</td>
<td>0.739</td>
</tr>
<tr>
<td><strong>Financial Conditions Predictors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banking crisis</td>
<td>Banking crisis</td>
<td>0.090</td>
<td>0.286</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Currency crisis</td>
<td>Exchange rate depreciation ≥ 15%</td>
<td>0.144</td>
<td>0.351</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Sovereign default</td>
<td>Sovereign default</td>
<td>0.095</td>
<td>0.293</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Hyperinflation</td>
<td>Annual inflation rate ≥ 20%</td>
<td>0.100</td>
<td>0.300</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Credit growth (world)</td>
<td>GDP-weighted 3-year change in total bank loans to GDP</td>
<td>0.019</td>
<td>0.042</td>
<td>−0.132</td>
<td>0.084</td>
</tr>
<tr>
<td><strong>Natural Disaster Predictors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural disaster</td>
<td>Major earthquake, tsunami, volcano eruption, and the Great Influenza</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Famine</td>
<td>Major famine</td>
<td>0.008</td>
<td>0.092</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td><strong>Asset Price Predictors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dividend-price ratio (US)</td>
<td>S&amp;P 500 dividend-price ratio</td>
<td>0.041</td>
<td>0.016</td>
<td>0.011</td>
<td>0.093</td>
</tr>
<tr>
<td>Stock volatility (US)</td>
<td>S&amp;P 500 realized volatility</td>
<td>0.205</td>
<td>0.072</td>
<td>0.054</td>
<td>0.513</td>
</tr>
<tr>
<td>Yield curve (US)</td>
<td>10-year–1-year Treasury spread</td>
<td>0.007</td>
<td>0.017</td>
<td>−0.050</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Notes. We report descriptive statistics for the predictive variables (see Section I.C and the Online Appendix for detailed descriptions of each variable).
Table II: Predicting Macroeconomic Crises: Saturated Specification Results

<table>
<thead>
<tr>
<th>Dependent variable: Indicator = 1 if macroeconomic crisis in year t + 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Crisis</td>
</tr>
<tr>
<td>Recession</td>
</tr>
<tr>
<td>Crisis abroad</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Consumption (world)</td>
</tr>
<tr>
<td>War</td>
</tr>
<tr>
<td>Civil war</td>
</tr>
<tr>
<td>Political crisis</td>
</tr>
<tr>
<td>War/political crisis abroad</td>
</tr>
<tr>
<td>Banking crisis</td>
</tr>
<tr>
<td>Currency crisis</td>
</tr>
<tr>
<td>Sovereign default</td>
</tr>
<tr>
<td>Hyperinflation</td>
</tr>
<tr>
<td>Credit growth (world)</td>
</tr>
<tr>
<td>Natural disaster</td>
</tr>
<tr>
<td>Famine</td>
</tr>
<tr>
<td>Dividend-price ratio (US)</td>
</tr>
<tr>
<td>Stock volatility (US)</td>
</tr>
<tr>
<td>Yield curve (US)</td>
</tr>
<tr>
<td>N</td>
</tr>
<tr>
<td>Countries</td>
</tr>
<tr>
<td>Crises</td>
</tr>
<tr>
<td>Pseudo R²</td>
</tr>
<tr>
<td>AUROC</td>
</tr>
<tr>
<td>σ(π̂)</td>
</tr>
<tr>
<td>ρ(π̂, π̂′)</td>
</tr>
</tbody>
</table>

Notes. This table shows standardized marginal effects from the probit model specified in Eq. (3). Observations are over the sample of 42 countries, 1876-2015. The dependent variable is equal to one when there is a macroeconomic disaster in country i at time t + 1. In the first five columns, a crisis is defined as a 2 standard-deviation drop in consumption growth, and 0 otherwise. Results are shown for the full sample, as well as subsamples comparing pre- and post-WW2 data, OECD and non-OECD countries. The next five columns report results with different crisis definitions. “Fixed” defines crises assuming consumption volatility stays constant after 1945; the next columns consider different crisis cutoffs: 1.5, 2.5, and 3 standard deviations from average consumption growth rate, instead of the 2-SD cutoff in Eq. (2). Finally, “GDP” corresponds to GDP crises. The table reports the AUROC statistics, the volatility of the macro estimates, as well as their pairwise correlations with the baseline model estimates. Standard errors are clustered by country and year.

, *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.
Table III: Out-of-sample Crisis Predictability

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Crises</th>
<th>In sample</th>
<th>Out of sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline disasters</td>
<td>3.621</td>
<td>134</td>
<td>0.816**</td>
<td>0.775***</td>
</tr>
<tr>
<td>Real-time disasters</td>
<td>3.245</td>
<td>65</td>
<td>0.865***</td>
<td>0.776***</td>
</tr>
<tr>
<td>Cross-validation</td>
<td>2.337</td>
<td>91</td>
<td>0.814***</td>
<td>0.772***</td>
</tr>
</tbody>
</table>

Notes. This table reports Area under the Receiver Operating Characteristic (AUROC) statistics for in-sample and out-of-sample macro risk estimates. The AUROC equals the area under the ROC curve, which measures the accuracy of a predictive signal by comparing the true positive rate (signal ratio) against the false positive rate (noise ratio) for every possible threshold value (see Figure A.III), with 0.5 corresponding to no explanatory power and 1 to a perfect fit. In-sample and out-of-sample estimates are compared over the 1922-2015 forecasting period. In-sample \( \hat{\pi} \) are estimated over the same sample period. Out-of-sample \( \hat{\pi} \) are constructed by fitting Eq. (3) on an expanding window after a 50-year training period (1876-1921). The first row shows forecasting results with crises identified in-sample. The second row reports results where crises are identified out-of-sample, based on consumption means and standard deviations estimated over the same expanding window as out-of-sample forecasts. The third row shows averaged cross-validation results were we repeatedly estimate the model on a random 21-country sample and test the model on the remaining 21 countries. We report statistical significance based on bootstrapped AUROC confidence bands.

*, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.
Table IV: Macro Risk and the Dividend-price Ratio Weakly Forecast Consumption Growth

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H=1 3 5</td>
<td>H=1 3 5</td>
<td>H=1 3 5</td>
</tr>
<tr>
<td></td>
<td>Consumption growth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>-0.19*** -0.20** 0.02</td>
<td>-0.17*** -0.22 -0.02</td>
<td>-0.12 0.25 0.28</td>
</tr>
<tr>
<td></td>
<td>(0.04) (0.12) (0.14)</td>
<td>(0.04) (0.13) (0.20)</td>
<td>(0.09) (0.15) (0.21)</td>
</tr>
<tr>
<td>N</td>
<td>4,695 4,603 4,511</td>
<td>1,826 1,754 1,682</td>
<td>2,869 2,849 2,829</td>
</tr>
<tr>
<td>R²</td>
<td>0.029 0.008 -0.000</td>
<td>0.023 0.013 -0.001</td>
<td>0.006 0.010 0.012</td>
</tr>
<tr>
<td></td>
<td>GDP growth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>-0.21*** -0.32*** -0.21</td>
<td>-0.18*** -0.24** 0.01</td>
<td>-0.13* -0.26 -0.31</td>
</tr>
<tr>
<td></td>
<td>(0.04) (0.10) (0.15)</td>
<td>(0.05) (0.11) (0.18)</td>
<td>(0.07) (0.24) (0.23)</td>
</tr>
<tr>
<td>N</td>
<td>4,686 4,592 4,498</td>
<td>1,824 1,752 1,680</td>
<td>2,862 2,840 2,818</td>
</tr>
<tr>
<td>R²</td>
<td>0.037 0.023 0.006</td>
<td>0.030 0.017 -0.001</td>
<td>0.008 0.009 0.013</td>
</tr>
<tr>
<td></td>
<td>Crises</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.92*** 1.84*** 2.36***</td>
<td>0.84*** 1.78*** 2.22***</td>
<td>1.17*** 1.61*** 2.28***</td>
</tr>
<tr>
<td></td>
<td>(0.10) (0.30) (0.42)</td>
<td>(0.11) (0.45) (0.73)</td>
<td>(0.22) (0.33) (0.39)</td>
</tr>
<tr>
<td>N</td>
<td>4,695 4,603 4,511</td>
<td>1,826 1,754 1,682</td>
<td>2,869 2,849 2,829</td>
</tr>
<tr>
<td>R²</td>
<td>0.070 0.076 0.071</td>
<td>0.073 0.084 0.059</td>
<td>0.047 0.039 0.073</td>
</tr>
</tbody>
</table>

Notes. This table presents slope coefficients, standard errors, and adjusted $R^2$ statistics for predictive panel regressions of cumulated consumption growth on crisis probabilities and the dividend-price ratio (DP). Observations are over the sample of 42 countries, 1876-2015. Results are reported for the full sample as well as for subsamples covering (respectively) the pre- and post-WW2 periods. Standard errors in parentheses are dually clustered on country and time.

*, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.
Table V: Macro Risk Forecasts Excess Stock Returns

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>H</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>b</td>
<td>0.86*** 1.98*** 3.21*** 0.43** 0.96*** 1.48*** 2.35*** 3.68*** 4.25***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.27) (0.65) (0.96) (0.17) (0.33) (0.37) (0.58) (0.96) (1.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1,663 1,581 1,501 164 150 137 1,499 1,431 1,364</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.014 0.025 0.040 0.045 0.096 0.137 0.028 0.032 0.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.74*** 1.50*** 2.70*** 0.40* 0.84** 1.14*** 1.56*** 2.14*** 3.18***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.25) (0.56) (0.81) (0.20) (0.33) (0.32) (0.60) (0.82) (0.79)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1,663 1,581 1,501 164 150 137 1,499 1,431 1,364</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.011 0.016 0.031 0.038 0.069 0.061 0.017 0.014 0.033</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.18** 0.24 0.31 0.17* 0.29*** 0.41*** 0.19* 0.19 0.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09) (0.16) (0.21) (0.08) (0.11) (0.06) (0.11) (0.16) (0.26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1,666 1,584 1,504 167 153 140 1,499 1,431 1,364</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.007 0.003 0.003 0.043 0.048 0.058 0.005 0.001 0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.06 -0.16 -0.41 0.09 -0.03 -0.07 -0.02 -0.26 -0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.12) (0.17) (0.27) (0.08) (0.13) (0.18) (0.14) (0.21) (0.33)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1,663 1,581 1,501 164 150 137 1,499 1,431 1,364</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.014 0.025 0.044 0.045 0.090 0.131 0.027 0.034 0.052</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. This table presents slope coefficients, standard errors, and adjusted \( R^2 \) statistics for predictive panel regressions of cumulated excess returns on crisis probabilities and the dividend-price ratio (DP). Observations are over the sample of 42 countries, 1876-2015. Results are reported for the full sample as well as for subsamples covering (respectively) the pre- and post-WW2 periods. Standard errors in parentheses are dually clustered on country and time. *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.
### Table VI: Estimation of the Disaster Risk Model

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>S.E.</th>
<th>95% Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Normal Times:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average growth in consumption $\mu$</td>
<td>0.021</td>
<td>0.002</td>
<td>(0.017, 0.025)</td>
</tr>
<tr>
<td>Volatility of consumption growth $\sigma$</td>
<td>0.017</td>
<td>0.001</td>
<td>(0.015, 0.021)</td>
</tr>
<tr>
<td><strong>Disaster probability:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-term disaster probability $\bar{\pi}$</td>
<td>0.039</td>
<td>0.003</td>
<td>(0.033, 0.044)</td>
</tr>
<tr>
<td>Persistence $\rho$</td>
<td>0.661</td>
<td>0.060</td>
<td>(0.563, 0.758)</td>
</tr>
<tr>
<td>Volatility parameter $\nu$</td>
<td>0.175</td>
<td>0.002</td>
<td>(0.172, 0.179)</td>
</tr>
<tr>
<td><strong>Disaster size:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape parameter $\alpha$</td>
<td>1.787</td>
<td>0.212</td>
<td>(1.441, 2.134)</td>
</tr>
<tr>
<td>Scale parameter $\beta$</td>
<td>0.073</td>
<td>0.009</td>
<td>(0.058, 0.088)</td>
</tr>
<tr>
<td>Shifting parameter $\theta$</td>
<td>0.017</td>
<td>0.002</td>
<td>(0.013, 0.020)</td>
</tr>
<tr>
<td>$\text{E}(v_t) = \alpha \beta + \theta$</td>
<td>0.147</td>
<td>0.007</td>
<td>(0.135, 0.159)</td>
</tr>
<tr>
<td>$\text{SD}(v_t) = \sqrt{\alpha \beta^2}$</td>
<td>0.098</td>
<td>0.007</td>
<td>(0.086, 0.110)</td>
</tr>
</tbody>
</table>

**Notes.** This table presents estimates of the time-varying disaster probability model. Log consumption growth evolves according to

$$\Delta c_t = \mu + \sigma \epsilon_t + v_t,$$

where $\mu$ and $\sigma$ are constants, $\epsilon_t$ is a standard normal random variable, $v_t = J_t \Delta n_t > 0$, and $\Delta n_t$ follows a Poisson distribution with time-varying probability $\pi_t$:

$$\pi_t - \bar{\pi} = \rho(\pi_{t-1} - \bar{\pi}) + \nu \sqrt{\pi_{t-1}} u_t.$$

Finally, $J_t$ follows a shifted negative gamma distribution that takes a minimum value (in magnitude) of $\theta$ and gamma parameters $\alpha$ and $\beta$. The log consumption growth mean $\mu$ and variance $\sigma$ are estimated from U.S. postwar data. The disaster probability parameters are recovered from the macro risk estimates excluding asset prices, $\tilde{\pi}$, in the panel regression (20), where standard errors are dually clustered on country and time. Size parameters are estimated via maximum likelihood from the sample of realized crises. Standard errors for the disaster size moments are obtained using the delta method.
Table VII: Asset Pricing Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Model</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1871-2015</td>
<td>1946-2015</td>
<td>2.5% 5% 50% 95%</td>
<td>97.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel A: Unrestricted Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average risk-free rate</td>
<td>1.92</td>
<td>0.37</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.36</td>
<td>0.63</td>
<td>0.68</td>
</tr>
<tr>
<td>Standard deviation of risk-free rate</td>
<td>4.69</td>
<td>2.80</td>
<td>2.13</td>
<td>2.18</td>
<td>2.49</td>
<td>2.86</td>
<td>2.95</td>
</tr>
<tr>
<td>Average excess return</td>
<td>6.27</td>
<td>7.77</td>
<td>4.97</td>
<td>5.07</td>
<td>5.66</td>
<td>6.18</td>
<td>6.28</td>
</tr>
<tr>
<td>Standard deviation of excess return</td>
<td>18.6</td>
<td>16.8</td>
<td>11.4</td>
<td>11.8</td>
<td>13.4</td>
<td>15.1</td>
<td>15.6</td>
</tr>
<tr>
<td>Average dividend yield</td>
<td>4.23</td>
<td>3.39</td>
<td>2.05</td>
<td>2.05</td>
<td>2.08</td>
<td>2.12</td>
<td>2.13</td>
</tr>
<tr>
<td>Standard deviation of dividend yield</td>
<td>1.58</td>
<td>1.42</td>
<td>0.23</td>
<td>0.24</td>
<td>0.28</td>
<td>0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>Autocorrelation of dividend yield</td>
<td>79.0</td>
<td>90.1</td>
<td>57.5</td>
<td>58.6</td>
<td>65.3</td>
<td>72.0</td>
<td>73.1</td>
</tr>
<tr>
<td><strong>Panel B: Restricted Model (No Realized Disasters)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average risk-free rate</td>
<td>1.92</td>
<td>0.37</td>
<td>-0.00</td>
<td>0.05</td>
<td>0.37</td>
<td>0.65</td>
<td>0.69</td>
</tr>
<tr>
<td>Standard deviation of risk-free rate</td>
<td>4.69</td>
<td>2.80</td>
<td>2.12</td>
<td>2.18</td>
<td>2.48</td>
<td>2.87</td>
<td>2.93</td>
</tr>
<tr>
<td>Average excess return</td>
<td>6.27</td>
<td>7.77</td>
<td>6.69</td>
<td>6.77</td>
<td>7.15</td>
<td>7.56</td>
<td>7.63</td>
</tr>
<tr>
<td>Standard deviation of excess return</td>
<td>18.6</td>
<td>16.8</td>
<td>9.3</td>
<td>9.4</td>
<td>10.1</td>
<td>10.8</td>
<td>11.0</td>
</tr>
<tr>
<td>Average dividend yield</td>
<td>4.23</td>
<td>3.39</td>
<td>2.05</td>
<td>2.05</td>
<td>2.08</td>
<td>2.12</td>
<td>2.12</td>
</tr>
<tr>
<td>Standard deviation of dividend yield</td>
<td>1.58</td>
<td>1.42</td>
<td>0.23</td>
<td>0.24</td>
<td>0.28</td>
<td>0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>Autocorrelation of dividend yield</td>
<td>79.0</td>
<td>90.1</td>
<td>56.8</td>
<td>57.9</td>
<td>64.8</td>
<td>71.9</td>
<td>73.1</td>
</tr>
</tbody>
</table>

Notes. Panel A reports unconditional moment statistics from S&P 500 real returns and three-month U.S. Treasury real rates as well as percentiles of the same moments from model simulations. Panel B reports the same quantities from a restricted model in which disasters do not realize. Parameters are from Table VI. Additional parameters are $\gamma = 6, \delta = 99\%$ and $\phi = 2.6$. 

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Table VIII: Asset Pricing Moments: Model Extensions

<table>
<thead>
<tr>
<th></th>
<th>Data 1871-2015</th>
<th>Data 1946-2015</th>
<th>CRRA 2.5%</th>
<th>CRRA 5%</th>
<th>CRRA 50%</th>
<th>CRRA 95%</th>
<th>CRRA 97.5%</th>
<th>Co-Integration 2.5%</th>
<th>Co-Integration 5%</th>
<th>Co-Integration 50%</th>
<th>Co-Integration 95%</th>
<th>Co-Integration 97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: The CRRA Case</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average risk-free rate</td>
<td>1.92</td>
<td>0.37</td>
<td>3.19</td>
<td>3.40</td>
<td>4.54</td>
<td>5.51</td>
<td>5.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of risk-free rate</td>
<td>4.69</td>
<td>2.80</td>
<td>7.50</td>
<td>7.79</td>
<td>8.93</td>
<td>10.49</td>
<td>10.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average excess return</td>
<td>6.27</td>
<td>7.77</td>
<td>3.19</td>
<td>3.45</td>
<td>4.40</td>
<td>5.46</td>
<td>5.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of excess return</td>
<td>18.6</td>
<td>16.8</td>
<td>15.0</td>
<td>15.3</td>
<td>16.9</td>
<td>18.8</td>
<td>19.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average dividend yield</td>
<td>4.23</td>
<td>3.39</td>
<td>4.72</td>
<td>4.72</td>
<td>4.78</td>
<td>4.83</td>
<td>4.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation of dividend yield</td>
<td>1.58</td>
<td>1.42</td>
<td>0.37</td>
<td>0.38</td>
<td>0.43</td>
<td>0.49</td>
<td>0.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autocorrelation of dividend yield</td>
<td>79.0</td>
<td>90.1</td>
<td>57.2</td>
<td>58.6</td>
<td>65.0</td>
<td>70.9</td>
<td>71.9</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

| **Panel B: The Co-Integration Case** |                |                |           |         |          |          |            |                     |                   |                    |                   |                    |
| Average risk-free rate         | 1.92           | 0.37           | 0.01      | 0.05    | 0.35     | 0.63     | 0.67       |                     |                   |                    |                   |                    |
| Standard deviation of risk-free rate | 4.69     | 2.80           | 2.13      | 2.19    | 2.50     | 2.88     | 2.98       |                     |                   |                    |                   |                    |
| Average excess return          | 6.27           | 7.77           | 4.50      | 4.58    | 5.16     | 5.75     | 5.88       |                     |                   |                    |                   |                    |
| Standard deviation of excess return | 18.6    | 16.8           | 13.7      | 13.9    | 15.1     | 16.7     | 17.0       |                     |                   |                    |                   |                    |
| Average dividend growth        | 3.58           | 5.98           | 1.14      | 1.21    | 1.47     | 1.70     | 1.75       |                     |                   |                    |                   |                    |
| Standard deviation of dividend growth | 13.0    | 7.7            | 17.5      | 17.8    | 19.5     | 21.8     | 22.2       |                     |                   |                    |                   |                    |
| Average dividend yield         | 4.23           | 3.39           | 3.62      | 3.68    | 4.12     | 4.59     | 4.71       |                     |                   |                    |                   |                    |
| Standard deviation of dividend yield | 1.58     | 1.42           | 0.96      | 1.00    | 1.24     | 1.54     | 1.62       |                     |                   |                    |                   |                    |
| Autocorrelation of dividend yield | 79.0   | 90.1           | 86.1      | 86.9    | 90.7     | 93.6     | 94.2       |                     |                   |                    |                   |                    |
| Corr. div. yield and disaster probability | 38.6   | 43             | 21.5      | 24.4    | 37.4     | 51.1     | 52.5       |                     |                   |                    |                   |                    |

Notes. Panel A reports unconditional moment statistics from S&P 500 real returns and three-month U.S. Treasury real rates as well as percentiles of the same moments from model simulations of our CRRA model extension. Parameters are from Table VI. Additional parameters are $\gamma = 6, \delta = 99\%$ and $\phi = 2.6$. Panel B reports the same quantities from model simulations of our cointegration model extension. Parameters are from Table VI. Additional parameters are $\gamma = 6, \delta = 99\%, \phi = 0.97, \eta = 0.15$, and $\kappa = 2.6$. 
Figure 1: U.S. Macroeconomic risk
Notes. This figure shows the one-year ahead probability of a severe macroeconomic crisis, $\hat{\pi}$, in the United States. A crisis is defined as a two standard deviation drop in consumption below the average growth rate. According to our definition, the United States experienced crises in 1921, 1930, and 1932.
Figure 2: Timeline of Macroeconomic Crises

Notes. Macroeconomic crises are defined as two-standard-deviation declines in consumption growth below their long-term growth rate. Small black dots indicate data unavailability.
Figure 3: Predicting Macroeconomic Crises

Notes. This figure shows marginal effects and 90% confidence bands from univariate versions of the probit model specified in Eq. (3). The marginal effects are standardized to represent the effect of a one standard deviation increase in the variable of interest on the likelihood of a macroeconomic crisis. Observations are over the sample of 42 countries, 1876-2015. Our panel consists of 4,695 observations of which 186 are macroeconomic disasters. The dependent variable is an indicator equal to one when there is a macroeconomic disaster in country $i$ at time $t + 1$, defined as a 2 standard-deviation drop in consumption growth, and 0 otherwise.
Figure 4: Country Crisis Probabilities, $\hat{\pi}_i$

Notes. Country crisis $\hat{\pi}$ estimates. Shaded areas indicate realized crises.
Figure 5: Comparison with Option-Based Estimates

Notes. This figure compares macro risk (blue) and option-based estimates (black lines), which are constructed by Barro and Liao (2020) using over-the-counter options prices for the following equity-market indices: S&P 500 (U.S.), FTSE (U.K.), DAX (Germany), Nikkei (Japan), OMX (Sweden), and SMI (Switzerland).
**Figure 6: Macro Risk and the Equity Premium**

*Notes.* This figure plots the U.S. macro risk estimates constructed without asset-price information, $\hat{\pi}^-$, against two equity premium proxies: the S&P 500 dividend-price ratio and Martin (2017)'s bound on the equity premium, $SVIX^2$. We extend the latter to our longer historical sample substituting our realized volatility index squared for the risk-neutral volatility in the true index, multiplied by the real risk-free rate.
Figure 7: Macroeconomic Crises and Stock Returns

Notes. This figure reports the local-currency average excess stock return around a macroeconomic crisis, obtained as the regression coefficients in Eq. (6).

Figure 8: Model-Implied Predictability

Notes. This figure shows predictive slopes from the regression of either future cumulative excess returns (left) or consumption growth (right) on either actual and model-implied macro risk as a function of the horizon. Model-implied slopes are the median from simulations of both the baseline model (we also report the 5th and 95th percentiles) and the restricted model in which disasters do not realize. Slopes with actual data correspond to point estimates from panel regressions (Tables IV and V).
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Appendix A. Data Appendix

Geographic distances. Our “Disaster abroad” variable is distance-weighted. We measure the distance between two countries as the population-weighted average distance in kilometers between large cities’ of each country pair (Mayer and Zignago, 2011).

GDP. We primarily work with consumption and GDP expressed per capita, but we sometimes need GDP levels to value-weight series. We thus obtain population data from the Maddison database (Inklaar et al., 2018), with the exception of pre-1950 Icelandic population data, which we downloaded from populstat.info (accessed on November 29, 2019). We interpolate the remaining small portion of the data that is missing (Shape-preserving piecewise cubic interpolation).

Wars. The data for war and civil wars come from Sarkees and Wayman (2010), which extends from 1816-2007. “Wars” correspond to the events listed as interstate wars, while “Civil wars” correspond to the events classified as either instate or non-state wars. We use Wikipedia pages listing wars involving each country in our sample to extend the data from 2008 to 2015.\footnote{E.g., \url{https://en.wikipedia.org/wiki/List_of_wars_involving_Argentina} (as of November 28, 2017). This yields a single new civil war: the Sinai Insurgency (Egypt, 2011-2015), and extended periods for two other civil wars, namely the Second Sri Lanka Tamil (Sri Lanka, 2006-2009), as well as unrest in Colombia (1989-2015).}

Political crises. We use data from the Center for Systemic Peace (CSP): Integrated Network for Societal Conflict Research. The constraint on the executive variable is constructed by the Polity IV project by coding the authority characteristics of governments across the globe from 1800 to 2016. The scale ranges from 1 (weak constraints on executive power) to 7 (strong constraints on executive power). The variable measures the extent of institutional constraints on executive power. We define political crises as a four or more decline in constraints on executive, or a collapse or interruption in authority (coded separately as “-66” and “-77” in the Polity IV dataset).

Sovereign defaults. We utilize external default dummies assembled by Carmen Reinhart and Kenneth Rogoff (RR, see Reinhart and Rogoff (2009)). The data covers the period 1800-2014. We use Wikipedia’s list of sovereign debt crises to complete our data up to 2015.\footnote{Accessed on November 28, 2017, at \url{https://en.wikipedia.org/wiki/List_of_sovereign_debt_crises}. In our sample of countries, we find one default in 2015 (Greece).}

Hyperinflation. Our hyperinflation dates primarily come from RR, who define inflation crises as periods with annual inflation rate exceeding 20%. The data covers the period 1800-2010. We extend the sample to 2015 using inflation data from the World Development Indicators as well as Global Financial Data.

Currency crises. We use currency crises dates from RR, which are defined as annual depreciation against the U.S. dollar (or the relevant anchor currency) of 15 percent or more. The data covers the period 1800-2010; we use exchange rate data from WDI and the Federal Reserve Bank of St. Louis to calculate crisis dates until 2015.

Financial crises. We use banking crisis dates constructed by RR, which covers the period 1800-2010. Banking crises correspond to events characterized by substantial bank runs and closure, merging, takeover, or large-scale government assistance of an important financial institution. We use Wikipedia’s list of banking crises to complete our data to 2015.\footnote{Accessed on November 28, 2017, at \url{https://en.wikipedia.org/wiki/List_of_banking_crises}. In our sample of countries, we find one default in 2015 (Greece). We find no evidence of sovereign defaults for Iceland.
Natural Disasters. We obtain data on international earthquakes, tsunamis, and volcano eruptions from the National Centers for Environmental Information. We focus on major events, the highest grade in event classifications. This corresponds to events that have caused at least 1,000 deaths, destroyed at least 1,000 houses or equivalent damages in monetary terms. When the database does not report precise numbers, we use the highest descriptive score (4), which corresponds to natural disasters of commensurable intensity. We also obtain the dates and location of major famines from the website Our World in Data (accessed on November 11, 2019).

Financial data. The data come from Global Financial Data (GFD). We compute local currency excess returns by subtracting the continuously compounded 3-month short-term interest rate from the total equity return (in logs). Returns are calculated from the end of the previous year to the end of the current year. To eliminate the influence of outliers, we drop the 99.5th and 0.5th percentile of stock excess returns. The former is GFD’s country Treasury Bill Yields. Our world variables are based on U.S. series, which have the longest time coverage. Our baseline global dividend-price ratio is the Standard & Poor’s 500 D/P ratio computed by GFD, which starts in 1871. Our volatility index is a composite of multiple time series. From 1986 onward, we use the CBOE VXO index of percentage implied volatility, on a hypothetical at the money S&P100 option 30 days to expiration. Following Bloom (2009), we use actual returns volatilities to extend the volatility series back in time. We estimate the annual standard deviation of stock returns using the daily returns to the Standard and Poor’s S&P 500 from 1928 through 1985. The estimates from 1885 through 1927 use daily returns on the Dow Jones composite portfolio, as in Schwert (1989). Finally, since daily stock return data are not available before 1885, we compute the volatility through Schwert (1989)’s generalization of the Officer (1973) moving standard deviation estimator using monthly S&P 500 returns. The volatility indices are normalized to have the same mean and variance as the VXO index when they overlap from 1986 onward. Finally, we use the one-year interest rates in the United States from Robert Shiller to proxy for the short-term risk-free rate. The term spread subtracts the one-year interest rate to the 10-year Treasury bond yield.

Appendix B. Crisis events

Figure A.I shows the long-term growth rates and volatiles we use to identify crisis events. Table A.I lists the 186 crises in our sample. Figure A.II compares U.S. crises to crises identified according to Barro and Ursúa (2008).
Figure A.I: Long-Term Growth Rates And Volatility In Normal Times

This figure shows the long-term growth rates and standard deviations we use to identify crisis events. We use distinct standard deviations for the pre- and post-WW2 samples in our baseline specification. Pre-WW2 standard deviations are indicated with empty markers and post-WW2 are indicated with red markers. Some countries have very few observations in the prewar period. We use the full-sample standard deviation whenever we have less than 25 years of prewar data and the corresponding volatilities are reported with red markers.
Figure A.II: Comparison with Barro-Ursúa (2008) peak-to-trough macroeconomic crises.

Notes. Panel A shows U.S. consumption growth together with the U.S. crisis cutoff value (red dashed line). U.S. crisis years correspond to the years in which a negative growth rate exceeds the the U.S. crisis cutoff value. Panel B shows U.S. log consumption. U.S. crisis years are highlighted in red. Shaded area illustrate U.S. crises according to Barro-Ursúa, which are defined as cumulative consumption declines exceeding 10%.
<table>
<thead>
<tr>
<th>Year</th>
<th>Disasters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1875</td>
<td>Denmark (0.09)</td>
</tr>
<tr>
<td>1877</td>
<td>Switzerland (0.16)</td>
</tr>
<tr>
<td>1878</td>
<td>Norway (0.06)</td>
</tr>
<tr>
<td>1879</td>
<td>Turkey (0.26)</td>
</tr>
<tr>
<td>1882</td>
<td>Switzerland (0.15)</td>
</tr>
<tr>
<td>1886</td>
<td>Switzerland (0.15)</td>
</tr>
<tr>
<td>1888</td>
<td>Switzerland (0.17)</td>
</tr>
<tr>
<td>1897</td>
<td>New Zealand (0.28)</td>
</tr>
<tr>
<td>1900</td>
<td>Argentina (0.22)</td>
</tr>
<tr>
<td>1905</td>
<td>Taiwan (0.18)</td>
</tr>
<tr>
<td>1908</td>
<td>Canada (0.12)</td>
</tr>
<tr>
<td>1913</td>
<td>Peru (0.06)</td>
</tr>
<tr>
<td>1914</td>
<td>Austria (0.32), Canada (0.11), Germany (0.19)</td>
</tr>
<tr>
<td>1915</td>
<td>Belgium (0.27), Chile (0.27), France (0.14), Turkey (0.35)</td>
</tr>
<tr>
<td>1916</td>
<td>Germany (0.11), Portugal (0.08), United Kingdom (0.09)</td>
</tr>
<tr>
<td>1917</td>
<td>Australia (0.15), Finland (0.14), Germany (0.13), Norway (0.10), Russia (0.23), Turkey (0.19), United Kingdom (0.08)</td>
</tr>
<tr>
<td>1918</td>
<td>Austria (0.27), Finland (0.27), Netherlands (0.47), Norway (0.08), Russia (0.38)</td>
</tr>
<tr>
<td>1919</td>
<td>Canada (0.13), Malaysia (0.20), Singapore (0.10)</td>
</tr>
<tr>
<td>1920</td>
<td>Egypt (0.16), Malaysia (0.21), Russia (0.27)</td>
</tr>
<tr>
<td>1921</td>
<td>Canada (0.18), Denmark (0.20), Norway (0.17), Sweden (0.14), United States (0.07)</td>
</tr>
<tr>
<td>1922</td>
<td>Chile (0.20), New Zealand (0.16)</td>
</tr>
<tr>
<td>1923</td>
<td>Germany (0.14)</td>
</tr>
<tr>
<td>1927</td>
<td>Chile (0.22)</td>
</tr>
<tr>
<td>1930</td>
<td>Colombia (0.18), Peru (0.11), United States (0.07)</td>
</tr>
<tr>
<td>1931</td>
<td>Australia (0.22), Chile (0.32), Italy (0.04), Malaysia (0.17), New Zealand (0.13), Singapore (0.09), Venezuela (0.23)</td>
</tr>
<tr>
<td>1932</td>
<td>Canada (0.10), Mexico (0.16), United States (0.10)</td>
</tr>
<tr>
<td>1933</td>
<td>India (0.05), Venezuela (0.24)</td>
</tr>
<tr>
<td>1936</td>
<td>Italy (0.04), Portugal (0.09), Spain (0.56)</td>
</tr>
<tr>
<td>1939</td>
<td>Greece (0.32)</td>
</tr>
<tr>
<td>1940</td>
<td>Belgium (0.29), Colombia (0.13), Denmark (0.26), Egypt (0.19), Finland (0.18), Greece (0.32), Japan (0.12), Netherlands (0.13), Sweden (0.08), United Kingdom (0.10)</td>
</tr>
<tr>
<td>1941</td>
<td>Belgium (0.40), France (0.37), Italy (0.06), Netherlands (0.12), United Kingdom (0.04)</td>
</tr>
<tr>
<td>1942</td>
<td>Colombia (0.12), France (0.23), Germany (0.11), Greece (0.24), Italy (0.14), Netherlands (0.38), Russia (0.55)</td>
</tr>
<tr>
<td>1943</td>
<td>Colombia (0.11), France (0.15), Italy (0.14), Japan (0.07), Korea (0.15), Netherlands (0.12)</td>
</tr>
<tr>
<td>1944</td>
<td>Japan (0.21), Korea (0.12), Taiwan (0.33), Turkey (0.18)</td>
</tr>
<tr>
<td>1945</td>
<td>Germany (0.20), Italy (0.09), Japan (0.43), Korea (0.20), Spain (0.14), Sweden (0.09), Taiwan (0.51)</td>
</tr>
<tr>
<td>1946</td>
<td>Belgium (0.07), Greece (0.12), India (0.11), New Zealand (0.07)</td>
</tr>
</tbody>
</table>

Notes. This table lists crises based on consumption data in our panel of 42 countries from 1876-2015. Disaster sizes are indicated in parentheses.
Appendix C. The Receiver Operating Characteristic Curve

The Receiver Operating Characteristic (ROC) curve is a standard tool used in binary classification problems, which has been recently used to assess early-warning signals of financial crises (e.g., Schularick and Taylor 2012). Figure A.III shows the curve corresponding to in-sample local crisis probabilities in the baseline model (Column 1 in Table II). In this type of setting, one is interested in converting probabilities into binary forecasts, which requires choosing a threshold over which to assign a value of one. The ROC curve shows the true positive rate (i.e., of all the crises that did happen, what fraction did the model predict?) against the false positive rate (i.e., how often the model wrongly predicts a macroeconomic crisis in $t + 1$?). The curve obtains by varying all possible threshold values. As the threshold increases, the number of crisis signals drops, so fewer crises are correctly identified and incorrectly signaled. In contrast, for a lower threshold, more crises are correctly identified; the cost is that the frequency of false signals also increases. A model with no forecasting power results in a 45-degree line, whereas a model with a perfect fit would have an elbow-shaped ROC running from (0,0) to (0,1) to (1,1). The goodness of fit is measured by the area under the ROC curve (AUROC), with 0.5 corresponding to no explanatory power and 1 to a perfect fit.

Figure A.III: Receiver Operating Characteristic (ROC) Curve

Notes. The ROC curve assess the accuracy of binary forecasts (in this case in-sample $\hat{\pi}_t$ estimates). Each dot corresponds to a given threshold rule, with darker dots representing higher thresholds. The x-axis indicates how often the model wrongly predict a macroeconomic crisis in $t + 1$; the y-axis gives the fraction of predicted disaster among all crises that did happen.
Appendix D. Macro Risk Estimates

Figure A.IV compares U.S. macro risk estimates and the cross-country average. Figure A.V plots macro risk estimates for sub-samples considered in Table II in the main text. Figure A.VI plots macro risk estimates for several alternative specification choices. Table A.II shows regression results for the linear probability model. Table A.III shows regression results for the logit model. Figure A.VI plots macro risk estimates for specifications that are less likely to be subject to look-ahead bias. Figure A.VIII compares the U.S. option-implied crisis probability in Barro and Liao (2019) together with the volatility index. The volatility index is an element in our predictor set (see Section I.C), which we show here scaled (using a Probit link) to be comparable with Barro and Liao’s option-implied probability.

![Figure A.IV: U.S. Estimates and Cross-Country Average](image)

**Figure A.IV: U.S. Estimates and Cross-Country Average**

**Notes.** This figure compares U.S. macro risk against the arithmetic average of individual country estimates.
This figure gives the macro risk estimates for the specifications reported in Table II in the main text. In the baseline model, a crisis is defined as a 2 standard-deviation drop in consumption growth, and 0 otherwise. We consider probabilities constructed with the full sample, as well as subsamples comparing pre- and post-WW2 data, OECD and non-OECD countries. We also consider different crisis cutoffs: 1.5, 2.5, and 3 standard deviations from average consumption growth rate. Finally, “GDP” corresponds to GDP crises and “Constant volatility” defines crises assuming consumption volatility stays constant after 1945.
Table A.II: Predicting Macroeconomic Crises: Saturated Specification Results (OLS)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline</th>
<th>Pre-1945</th>
<th>Post-1945</th>
<th>OECD</th>
<th>Ex-OECD</th>
<th>Fixed</th>
<th>1.5 SD</th>
<th>2.5 SD</th>
<th>3 SD</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crisis</td>
<td>0.015**</td>
<td>0.019*</td>
<td>0.011*</td>
<td>0.017**</td>
<td>0.010</td>
<td>0.012**</td>
<td>0.014**</td>
<td>0.012***</td>
<td>0.007**</td>
<td>0.018***</td>
</tr>
<tr>
<td>Recession</td>
<td>0.014***</td>
<td>0.007</td>
<td>0.022***</td>
<td>0.016***</td>
<td>0.010</td>
<td>0.011**</td>
<td>0.014**</td>
<td>0.007**</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Crisis abroad</td>
<td>0.004</td>
<td>-0.000</td>
<td>-0.002</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.001</td>
<td>0.007</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.007</td>
<td>0.004</td>
<td>0.013</td>
<td>0.009</td>
<td>0.004</td>
<td>0.006</td>
<td>0.007</td>
<td>0.005</td>
<td>0.001</td>
<td>-0.007</td>
</tr>
<tr>
<td>Consumption (world)</td>
<td>-0.001</td>
<td>0.000</td>
<td>0.009</td>
<td>0.000</td>
<td>-0.003</td>
<td>-0.008</td>
<td>-0.005</td>
<td>0.002</td>
<td>0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td>War</td>
<td>0.010*</td>
<td>0.019**</td>
<td>0.001</td>
<td>0.008</td>
<td>0.015</td>
<td>0.010</td>
<td>0.016**</td>
<td>0.008**</td>
<td>0.008**</td>
<td>0.010</td>
</tr>
<tr>
<td>Civil war</td>
<td>-0.001</td>
<td>-0.009</td>
<td>0.002</td>
<td>-0.006</td>
<td>0.001</td>
<td>0.003</td>
<td>0.001</td>
<td>-0.000</td>
<td>0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>Political crisis</td>
<td>0.005</td>
<td>0.002</td>
<td>0.007</td>
<td>0.003</td>
<td>0.013</td>
<td>0.010**</td>
<td>0.007</td>
<td>0.003</td>
<td>0.006</td>
<td>0.011***</td>
</tr>
<tr>
<td>War/political crisis abroad</td>
<td>0.018***</td>
<td>0.029**</td>
<td>0.006</td>
<td>0.021**</td>
<td>0.010</td>
<td>0.023***</td>
<td>0.19**</td>
<td>0.008*</td>
<td>0.009**</td>
<td>0.014*</td>
</tr>
<tr>
<td>Banking crisis</td>
<td>0.010**</td>
<td>0.007</td>
<td>0.013***</td>
<td>0.012**</td>
<td>0.007</td>
<td>0.006*</td>
<td>0.015***</td>
<td>0.004</td>
<td>0.002</td>
<td>0.009**</td>
</tr>
<tr>
<td>Currency crisis</td>
<td>0.011**</td>
<td>0.009</td>
<td>0.012**</td>
<td>0.012*</td>
<td>0.010</td>
<td>0.009**</td>
<td>0.013**</td>
<td>0.007**</td>
<td>0.003</td>
<td>0.013***</td>
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<tr>
<td>Sovereign default</td>
<td>-0.003</td>
<td>-0.001</td>
<td>-0.005</td>
<td>-0.000</td>
<td>-0.005</td>
<td>0.002</td>
<td>0.002</td>
<td>-0.004</td>
<td>-0.001</td>
<td>-0.000</td>
</tr>
<tr>
<td>Hyperinflation</td>
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<td>0.001</td>
<td>-0.003</td>
<td>0.005</td>
<td>-0.007</td>
<td>-0.006</td>
<td>0.006</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Credit growth (world)</td>
<td>-0.004</td>
<td>0.008</td>
<td>-0.003</td>
<td>-0.006</td>
<td>0.002</td>
<td>-0.007</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.004</td>
</tr>
<tr>
<td>Natural disaster</td>
<td>-0.003</td>
<td>0.004</td>
<td>-0.006**</td>
<td>-0.005</td>
<td>0.001</td>
<td>-0.002</td>
<td>-0.005</td>
<td>0.000</td>
<td>-0.001</td>
<td>-0.002</td>
</tr>
<tr>
<td>Famine</td>
<td>0.007*</td>
<td>0.007</td>
<td>0.004</td>
<td>0.010*</td>
<td>0.001</td>
<td>0.008*</td>
<td>0.007</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Dividend-price ratio (US)</td>
<td>0.011**</td>
<td>0.010</td>
<td>0.012**</td>
<td>0.010*</td>
<td>0.013*</td>
<td>0.014**</td>
<td>0.024***</td>
<td>0.009***</td>
<td>0.003</td>
<td>0.008</td>
</tr>
<tr>
<td>Stock volatility (US)</td>
<td>0.010***</td>
<td>0.001</td>
<td>0.016***</td>
<td>0.010**</td>
<td>0.010</td>
<td>0.012***</td>
<td>0.013**</td>
<td>0.007**</td>
<td>0.003*</td>
<td>0.015*</td>
</tr>
<tr>
<td>Yield curve (US)</td>
<td>0.002</td>
<td>0.012</td>
<td>-0.002</td>
<td>-0.001</td>
<td>0.008</td>
<td>-0.007</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>Constant</td>
<td>0.039***</td>
<td>0.056***</td>
<td>0.035***</td>
<td>0.040***</td>
<td>0.040***</td>
<td>0.044**</td>
<td>0.072***</td>
<td>0.023***</td>
<td>0.013***</td>
<td>0.042***</td>
</tr>
<tr>
<td>N</td>
<td>4,695</td>
<td>1,826</td>
<td>2,869</td>
<td>3,119</td>
<td>1,576</td>
<td>4,695</td>
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<td>4,695</td>
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<tr>
<td>Countries</td>
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<td>42</td>
<td>25</td>
<td>17</td>
<td>42</td>
<td>42</td>
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<tr>
<td>Crises</td>
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<td>109</td>
<td>77</td>
<td>126</td>
<td>60</td>
<td>210</td>
<td>346</td>
<td>108</td>
<td>64</td>
<td>201</td>
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<tr>
<td>$R^2$</td>
<td>0.067</td>
<td>0.090</td>
<td>0.048</td>
<td>0.087</td>
<td>0.036</td>
<td>0.079</td>
<td>0.063</td>
<td>0.050</td>
<td>0.045</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Notes. This table shows estimates from a linear probability model variant of the regression specified in Eq. (3). Baseline probit results are reported in Table II. Standard errors are clustered by country and year. *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.
Table A.III: Predicting Macroeconomic Crises: Saturated Specification Results (Logit)

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Pre-1945</th>
<th>Post-1945</th>
<th>OECD</th>
<th>Ex-OECD</th>
<th>Fixed</th>
<th>1.5 SD</th>
<th>2.5 SD</th>
<th>3 SD</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crisis</td>
<td>0.004***</td>
<td>0.006**</td>
<td>0.002**</td>
<td>0.004***</td>
<td>0.004</td>
<td>0.006**</td>
<td>0.002***</td>
<td>0.001**</td>
<td>0.005***</td>
<td></td>
</tr>
<tr>
<td>Recession</td>
<td>0.010***</td>
<td>0.006</td>
<td>0.009***</td>
<td>0.011***</td>
<td>0.009*</td>
<td>0.009***</td>
<td>0.013***</td>
<td>0.005***</td>
<td>0.002</td>
<td>0.004</td>
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<tr>
<td>Crisis abroad</td>
<td>−0.000</td>
<td>−0.002</td>
<td>−0.000</td>
<td>−0.000</td>
<td>0.001</td>
<td>−0.001</td>
<td>−0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>−0.001</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.005*</td>
<td>0.003</td>
<td>0.005**</td>
<td>0.005*</td>
<td>0.004</td>
<td>0.005**</td>
<td>0.007</td>
<td>0.003*</td>
<td>0.001</td>
<td>−0.001</td>
</tr>
<tr>
<td>Consumption (world)</td>
<td>0.002</td>
<td>−0.001</td>
<td>0.003</td>
<td>0.002</td>
<td>−0.002</td>
<td>−0.001</td>
<td>−0.000</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>War</td>
<td>0.003**</td>
<td>0.006**</td>
<td>0.000</td>
<td>0.002</td>
<td>0.005</td>
<td>0.002</td>
<td>0.007***</td>
<td>0.002**</td>
<td>0.001**</td>
<td>0.003</td>
</tr>
<tr>
<td>Civil war</td>
<td>−0.000</td>
<td>−0.006</td>
<td>0.001</td>
<td>−0.003</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>−0.000</td>
<td>0.000</td>
<td>−0.001</td>
</tr>
<tr>
<td>Political crisis</td>
<td>0.001</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.005</td>
<td>0.002*</td>
<td>0.002</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002**</td>
</tr>
<tr>
<td>War/political crisis</td>
<td>0.008***</td>
<td>0.013**</td>
<td>0.005**</td>
<td>0.007**</td>
<td>0.008*</td>
<td>0.010***</td>
<td>0.012**</td>
<td>0.003*</td>
<td>0.002*</td>
<td>0.008**</td>
</tr>
<tr>
<td>Banking crisis</td>
<td>0.006***</td>
<td>0.006</td>
<td>0.004***</td>
<td>0.006***</td>
<td>0.005</td>
<td>0.005**</td>
<td>0.011***</td>
<td>0.003**</td>
<td>0.001*</td>
<td>0.006***</td>
</tr>
<tr>
<td>Currency crisis</td>
<td>0.006***</td>
<td>0.006</td>
<td>0.005**</td>
<td>0.006***</td>
<td>0.007*</td>
<td>0.005**</td>
<td>0.010***</td>
<td>0.004***</td>
<td>0.002</td>
<td>0.007***</td>
</tr>
<tr>
<td>Sovereign default</td>
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<td>−0.000</td>
<td>−0.002</td>
<td>−0.000</td>
<td>−0.004</td>
<td>0.002</td>
<td>−0.002</td>
<td>−0.000</td>
<td>−0.000</td>
<td>0.001</td>
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<tr>
<td>Hyperinflation</td>
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<td>−0.001</td>
<td>0.002</td>
<td>−0.004</td>
<td>−0.002</td>
<td>0.004</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>Credit growth (world)</td>
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<td>0.004</td>
<td>−0.003</td>
<td>−0.004</td>
<td>0.002</td>
<td>−0.002</td>
<td>−0.001</td>
<td>−0.001</td>
<td>−0.001</td>
<td>−0.003</td>
</tr>
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<td>Natural disaster</td>
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<td>−0.004</td>
<td>−0.001</td>
<td>0.000</td>
<td>−0.002</td>
<td>−0.003</td>
<td>0.001</td>
<td>0.000</td>
<td>−0.000</td>
</tr>
<tr>
<td>Famine</td>
<td>0.002*</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002*</td>
<td>0.001</td>
<td>0.002*</td>
<td>0.003</td>
<td>0.001</td>
<td>0.000</td>
<td>−0.000</td>
</tr>
<tr>
<td>Dividend-price ratio (US)</td>
<td>0.008**</td>
<td>0.008</td>
<td>0.007***</td>
<td>0.007**</td>
<td>0.008*</td>
<td>0.010***</td>
<td>0.020***</td>
<td>0.006***</td>
<td>0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>Stock volatility (US)</td>
<td>0.006***</td>
<td>0.001</td>
<td>0.009***</td>
<td>0.005**</td>
<td>0.006</td>
<td>0.007***</td>
<td>0.009**</td>
<td>0.003**</td>
<td>0.002*</td>
<td>0.010**</td>
</tr>
<tr>
<td>Yield curve (US)</td>
<td>0.001</td>
<td>0.012</td>
<td>−0.001</td>
<td>−0.000</td>
<td>0.006</td>
<td>−0.005</td>
<td>0.000</td>
<td>0.001</td>
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</tr>
<tr>
<td>$N$</td>
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<td>1,826</td>
<td>2,869</td>
<td>3,119</td>
<td>1,576</td>
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<td>Crises</td>
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<td>126</td>
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<td>210</td>
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<td>108</td>
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<tr>
<td>Pseudo $R^2$</td>
<td>0.148</td>
<td>0.147</td>
<td>0.166</td>
<td>0.188</td>
<td>0.093</td>
<td>0.158</td>
<td>0.100</td>
<td>0.163</td>
<td>0.174</td>
<td>0.133</td>
</tr>
</tbody>
</table>

Notes. This table shows estimates from a Logit model variant of the regression specified in Eq. (3). Baseline probit results are reported in Table II. Standard errors are clustered by country and year.

*, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.
Panel A shows estimates excluding asset price predictors. Panel B shows estimates based on linear probability and logistic models, and estimates constructed from quantile regressions. Specifically, following Adrian et al. (2019), we estimate quantile regressions at the 1, 2, 3, 4, 5, 25, 50, 75, 95, and 99 percent quantiles, which we then smooth using the skewed $t$-distribution (Azzalini and Capitanio, 2003). To mimic our baseline specification, we work with standardized consumption, using distinct standard deviations in the prewar and postwar, and calculate the quantile probability from the $−2$-SD percentile. Panel C shows real-time estimates constructed in Section II.B. Panel D shows estimates constructed with peak-to-trough macroeconomic crises identified in Barro and Ursúa (2008).
Panel A shows estimates were wars and political crises, natural disasters, as well as the financial conditions crisis indicator are dropped. Panel B shows the probabilities of macroeconomic crises two and three years ahead.

Figure A.VII: Macro Risk Estimates: Robustness to Look-Ahead Bias
Panel A shows estimates were wars and political crises, natural disasters, as well as the financial conditions crisis indicator are dropped. Panel B shows the probabilities of macroeconomic crises two and three years ahead.
Figure A.VIII: Option-Based Estimates and the Volatility Index

Notes. This figure shows the U.S. option-implied crisis probability in Barro and Liao (2019) together with the volatility index.
Appendix E. Solving the Model

The dynamics of consumption belong to the affine class and are given by

\[ \Delta c_t = \mu + \sigma \xi_t + v_t, \]
\[ \pi_t = (1 - \rho) \bar{\pi} + \rho \pi_{t-1} + \nu \sqrt{\pi_{t-1}} u_t. \]

Therefore, the following expectation has exponential affine solution (Drechsler and Yaron, 2011):

\[ \mathbb{E}_t[e^{u_1 \Delta c_{t+1} + u_2 \pi_{t+1}}] = e^{g_0(u) + g_1(u)''[\Delta c_t, \pi_t]'}, \quad u = (u_1, u_2)' \in \mathbb{R}^2, \]

where

\[ g_0(u) = (\mu - \frac{\sigma^2}{2}) u_1 + \bar{\pi} (1 - \rho) u_2 + \frac{1}{2} \sigma^2 u_2^2, \]
\[ g_1(u) = [0, \frac{1}{2} \nu^2 u_2^2 + \rho u_2 + \varphi(u_1) - 1]'. \]

The representative agent has recursive utility of the form

\[ V_t = [(1 - \bar{\delta}) C_t^{1-1/\psi} + \delta (\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{1-1/\psi}]^{1/(1-1/\psi)}. \]

Normalized utility obtains if we take the limit as \( \psi \to 1 \), divide by \( C_t \), rearrange, and then take the logarithm:

\[ v_c t = \frac{\delta}{1 - \gamma} \log(\mathbb{E}_t[e^{(1-\gamma)[\Delta c_{t+1} + v_c \pi_{t+1}]}]). \]

Hypothesizing an affine form for \( v_c t \),

\[ v_c t = v_0 + v_c \Delta c_t + v_\pi \pi_t, \]

we can substitute to obtain

\[ v_0 + v_c \Delta c_t + v_\pi \pi_t = \frac{\delta}{1 - \gamma} \log(\mathbb{E}_t[e^{(1-\gamma)[\Delta c_{t+1} + v_0 + v_c \Delta c_{t+1} + v_\pi \pi_{t+1}]}]) \]

and then compute the expectation on the right-hand side:

\[ v_0 + v_c \Delta c_t + v_\pi \pi_t = \frac{\delta}{1 - \gamma} \left( (1 - \gamma) v_0 + g_0([(1 - \gamma)(1 + v_c), (1 - \gamma) v_\pi]') + g_1([(1 - \gamma)(1 + v_c), (1 - \gamma) v_\pi]')'[\Delta c_t, \pi_t]' \right). \]

Finally, we solve for the coefficients:

\[ v_c : v_c = 0, \]
\[ v_\pi : v_\pi = v_\pi \delta \rho + \delta (1 - \gamma) v_\pi^2 \nu^2 / 2 + \frac{\delta}{1 - \gamma} (\nu (1 - \gamma) - 1), \]
\[ v_0 : v_0 = v_0 \delta + \mu \delta + v_\pi \delta (1 - \rho) \bar{\pi} - \delta \gamma \sigma^2 / 2, \]

where we take the negative root to solve for \( v_\pi \).

In order to derive asset prices, we first solve for the stochastic discount factor:

\[ M_{t+1} = \delta e^{-\gamma \Delta c_{t+1} + (1 - \gamma) v_c t + (1 - \gamma) v_\pi t} \frac{\mathbb{E}_t[e^{(1-\gamma)[\Delta c_{t+1} + v_0 + v_c \Delta c_{t+1} + v_\pi \pi_{t+1}]}]}{\mathbb{E}_t[e^{(1-\gamma)[\Delta c_{t+1} + v_0 + v_c \Delta c_{t+1} + v_\pi \pi_{t+1}]}]} = \delta e^{-\gamma \Delta c_{t+1} + (1 - \gamma) v_c t + (1 - \gamma) v_\pi t} - g_0([(1 - \gamma)(1 - \gamma) v_\pi]') - g_1([(1 - \gamma)(1 - \gamma) v_\pi]'[\Delta c_t, \pi_t]'). \]

Since the risk-free rate satisfies \( e^{-r_{f,t}} = \mathbb{E}_t[M_{t+1}] \), we obtain

\[ e^{-r_{f,t}} = \delta e^{(1 - \gamma) v_0 + g_0([(1 - \gamma)(1 - \gamma) v_\pi]') + g_1([(1 - \gamma)(1 - \gamma) v_\pi]'[\Delta c_t, \pi_t]')} \times e^{-(1 - \gamma) v_0 - g_0([(1 - \gamma)(1 - \gamma) v_\pi]') - g_1([(1 - \gamma)(1 - \gamma) v_\pi]'[\Delta c_t, \pi_t]')} \]
Then we rearrange terms and solve the expectation as follows:

\[ r_{d,t} = -\log\delta - \mu - \gamma \sigma^2 + \pi_t(\varphi(1 - \gamma) - \varphi(-\gamma)). \]

Recall that dividends can be expressed as \( \Delta d_t = \phi \Delta c_t \). Stock returns are therefore given by

\[
\begin{align*}
    r_{d,t+1} &= \log \frac{P_{t+1}D_{t+1}}{P_tD_t} = \log \frac{P_{t+1}D_{t+1}+1}{P_tD_t} + \log \frac{D_{t+1}}{D_t} \\
    &= \log(e^{-d_{t+1}+p_{t+1}} + 1) + d_t - p_t + \Delta d_{t+1} \\
    &\approx k_0 - k_1(d_{t+1} - p_{t+1}) + d_t - p_t + \Delta d_{t+1}
\end{align*}
\]

for some endogenous constants \( k_0 \) and \( k_1 \) to be derived later. We posit an affine form for the logarithm of the D/P ratio:

\[
d_t - p_t = A_0 + A_c \Delta c_t + A_\pi \pi_t.
\]

The Euler equation for the stock (i.e., the claim asset on \( D_t \)) is

\[
1 = E_t[M_{t+1}e^{r_{d.t+1}}]
\]

We now plug in \( M_{t+1} \), the log-linearized \( r_{d.t+1} \), and our affine guesses for \( d_t - p_t \) and \( d_{t+1} - p_{t+1} \):

\[
1 = E_t[\delta e^{-\gamma \Delta c_{t+1} + (1-\gamma)\pi c_{t+1} - (1-\gamma)\pi_0 - g_0(1-\gamma,1-\gamma)\pi_0'\pi_t') | \Delta c_t, \pi_t']
\]

Then we rearrange terms and solve the expectation as follows:

\[
1 = e^{k_0 - k_1(A_0 + A_c \Delta c_t + A_\pi \pi_t)}(A_0 + A_c \Delta c_t + A_\pi \pi_t) + (A_0 + A_c \Delta c_t + A_\pi \pi_t) + \phi \Delta c_{t+1}].
\]

Finally, we solve for the coefficients of the D/P ratio and the log linearization constants:

\[
k_0 = -k_1 \log(k_1) - (1-k_1) \log(1-k_1),
\]

\[
\log k_1 = \log(1-k_1) - A_0 - A_c E[\Delta c_t] - A_\pi E[\pi_t].
\]

The coefficients satisfy

\[
A_c : \quad A_c = 0,
\]

\[
A_\pi : \quad A_\pi = g_1((1-\gamma,1-\gamma)\pi_0')[0,1]' - g_1((\phi - \gamma - k_1 A_c, (1-\gamma)\pi_0 - k_1 A_\pi')'[0,1]',
\]

\[
A_0 : \quad A_0 = -\log \delta - k_0 + k_1 A_0 - g_0((\phi - \gamma - k_1 A_c, (1-\gamma)\pi_0 - k_1 A_\pi')' + g_0((1-\gamma,1-\gamma)\pi_0').
\]

Note that one must solve simultaneously for \( k_1 \) and \( A_\pi \) (we take the positive root) and then for \( k_0 \) and \( A_0 \). Hence the equity premium is given by

\[
\text{log} E_t[e^{r_{d.t+1}}] - r_{f,t}
\]

\[
= \log(E_t[e^{r_{d.t+1}}]E_t[M_{t+1}])
\]

\[
= \log E_t[e^{r_{d.t+1}}] + \log E_t[M_{t+1}^J] - \log E_t[e^{r_{d.t+1}M_{t+1}^J} - \text{cov}_t[r_{d,t+1}, M_{t+1}^C]
\]

\[
= \langle \phi, -k_1 A_\pi \rangle \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\gamma(\gamma - 1)\pi_0'] + \pi_t[\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi - \gamma) - 1],
\]

where the superscripts \( C \) and \( J \) denote (respectively) the normal and non-normal components.

The return variance can be expressed as

\[
\text{var}_t[r_{d,t+1}] = \langle \phi, -k_1 A_\pi \rangle \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\phi, -k_1 A_\pi]' + \phi^2 \left( \frac{\partial^2 \varphi(u)}{\partial \pi_t^2} \right) \text{var}_t[\pi_t].
\]

Finally, the risk-neutral return variance satisfies

\[
\text{var}^Q_t[r_{d,t+1}] = \langle \phi, -k_1 A_\pi \rangle \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\phi, -k_1 A_\pi]' + \phi^2 \left( \frac{\partial^2 \varphi(u)}{\partial \pi_t^2} \right) \text{var}_t[\pi_t]
\]

(Drechsler and Yaron, 2011).
Appendix F. Model Extensions: the CRRA Case

Under CRRA preferences of the representative agent the stochastic discount factor simplifies to:

\[ M_{t+1} = \delta e^{-\gamma \Delta_c t}. \]

Since the risk-free rate satisfies \( e^{-r_{f,t}} = E_t[M_{t+1}] \), we obtain

\[ e^{-r_{f,t}} = \delta e^{g_0([-1,0]) + g_1([-1,0]) |\Delta \pi_t|} \]

and therefore

\[ r_{f,t} = -\log \delta + \gamma \mu - \gamma (1 + \gamma) \sigma^2 - \pi_t \varphi (1 - \gamma). \]

Recall that dividends can be expressed as \( \Delta d_t = \phi \Delta c_t \). Stock returns are therefore given by

\[ r_{d,t+1} = \log \frac{P_{t+1+D_{t+1}}}{P_t} = \log \frac{P_{t+1+D_{t+1}}}{P_t} + \log \frac{D_{t+1}}{D_t} \]

\[ \approx k_0 - k_1 (d_{t+1} - p_{t+1}) + d_t - p_t + \Delta d_{t+1} \]

for some endogenous constants \( k_0 \) and \( k_1 \) to be derived later. We posit an affine form for the logarithm of the D/P ratio:

\[ d_t - p_t = A_0 + A_c \Delta c_t + A_\pi \pi_t. \]

The Euler equation for the stock (i.e., the claim asset on \( D_t \)) is 1 = \( E_t[M_{t+1} e^{r_{d,t+1}}] \). We now plug in \( M_{t+1} \), the log-linearized \( r_{d,t+1} \), and our affine guesses for \( d_t - p_t \) and \( d_{t+1} - p_{t+1} \):

\[ 1 = E_t[\delta e^{-\gamma \Delta c_{t+1} - g_0([-1,0])' + k_0 - k_1 A_0 + (A_0 + A_c \Delta c_t + A_\pi \pi_t)}] \]

\[ \times e^{k_0 - k_1 (A_0 + A_c \Delta c_{t+1} + A_\pi \pi_{t+1}) + (A_0 + A_c \Delta c_t + A_\pi \pi_t) + \phi \Delta c_{t+1}}]. \]

Then we rearrange terms and solve the expectation as follows:

\[ 1 = \delta e^{-g_0([-1,0])' + g_1([-1,0])' |\Delta \pi_t| '} + k_0 - k_1 A_0 + (A_0 + A_c \Delta c_t + A_\pi \pi_t) \]

\[ \times E_t[e^{-\gamma \Delta c_{t+1} - k_1 (A_0 + A_c \Delta c_{t+1} + A_\pi \pi_{t+1}) + \phi \Delta c_{t+1}}] \]

\[ = \delta e^{-g_0([-1,0])' + g_1([-1,0])' |\Delta \pi_t| '} + k_0 - k_1 A_0 + (A_0 + A_c \Delta c_t + A_\pi \pi_t) \]

\[ \times e^{g_0([\phi - \gamma - k_1 A_c - k_1 A_\pi]') + g_1([\phi - \gamma - k_1 A_c - k_1 A_\pi]') |\Delta \pi_t| '}. \]

Finally, we solve for the coefficients of the D/P ratio and the log linearization constants:

\[ k_0 = -k_1 \log(k_1) - (1 - k_1) \log(1 - k_1), \]

\[ k_1 = \log(1 - k_1) - A_0 - A_c E[\Delta c_t] - A_\pi E[\pi_t]. \]

The coefficients satisfy

\[ A_c : A_c = 0, \]

\[ A_\pi : A_\pi = g_1([-1,0]') [0,1]' - g_1([\phi - \gamma - k_1 A_c, -k_1 A_\pi]') [0,1]', \]

\[ A_0 = -\log \delta - k_0 + k_1 A_0 - g_0([\phi - \gamma - k_1 A_c, -k_1 A_\pi]') + g_0([-1,0]'). \]

Note that one must solve simultaneously for \( k_1 \) and \( A_\pi \) (we take the positive root) and then for \( k_0 \) and \( A_0 \). Hence the equity premium is given by

\[ \log E_t[e^{r_{d,t+1}}] - r_{f,t} \]

\[ = \log(E_t[e^{r_{d,t+1}}]E_t[M_{t+1}]) \]

\[ = \log E_t[e^{r_{d,t+1}}] + \log E_t[M_{t+1}] - \log E_t[e^{r_{d,t+1}} M_{t+1}^J] - \text{cov} [r_{d,t+1}, M_{t+1}^C] \]

\[ = [\phi - k_1 A_\pi] [\sigma^2 \ 0 \ \nu^2 \pi_t] [\gamma,0]' + \pi_t [\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi-\gamma) - 1], \]

A17
we can substitute to obtain

\[
\text{var}_t[r_{t+1}'] = \left[\phi, -k_1 A_\pi\right] \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi \end{bmatrix} \left[\phi, -k_1 A_\pi\right]' + \phi^2 \left(\frac{\partial^2}{\partial \psi^2} \varphi(u)|_{u=0}\right) \pi_t. 
\]

Finally, the risk-neutral return variance satisfies

\[
\text{var}_t[\varphi_{t+1}'] = \left[\phi, -k_1 A_\pi\right] \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi \end{bmatrix} \left[\phi, -k_1 A_\pi\right]' + \phi^2 \left(\frac{\partial^2}{\partial \psi^2} \varphi(u)|_{u=-\gamma}\right) \pi_t
\]

(Drechsler and Yaron, 2011).

**Appendix G. Model Extensions: the Cointegration Case**

The dynamics of consumption and dividends belong to the affine class and are given by

\[
\Delta c_t = \mu + \sigma \varepsilon_t + \upsilon_t,
\]

\[
\Delta d_t = \Delta c_t + \Delta s_t,
\]

\[
s_t = (1 - \phi) s + \phi s_{t-1} + \eta z_t + \nu t,
\]

\[
\pi_t = (1 - \rho) \pi + \rho \pi_{t-1} + \nu \sqrt{\pi_{t-1}} u_t,
\]

where \(\varepsilon_t, z_t,\) and \(u_t\) and mutually independent standard normal random variables and

\[
v_t = J_t 1_{\Delta n_t > 0},
\]

\[
\nu_t = \kappa J_t 1_{\Delta n_t > 0},
\]

for \(\kappa > 0\). Therefore, the following expectation has exponential affine solution (Drechsler and Yaron, 2011):

\[
E_t[e^{u_1 \Delta c_{t+1} + u_2 \pi_{t+1} + u_3 s_{t+1}}] = e^{g_0(u) + g_1(u)'[\Delta c_t, \pi_t, s_t]'}, \quad u = (u_1, u_2, u_3)' \in \mathbb{R}^3,
\]

where

\[
g_0(u) = (\mu - \sigma^2) u_1 + \pi (1 - \rho) u_2 + \bar{s}(1 - \phi) u_3 + \frac{1}{2} \left(\sigma^2 u_1^2 + \eta^2 u_3^2\right),
\]

\[
g_1(u) = \left[0, \frac{1}{2} \sigma^2 u_2^2 + \rho u_2 + \varphi(u_1) + \varphi(\kappa u_3) - 2, \phi u_3\right]'.
\]

The representative agent has recursive utility of the form

\[
V_t = \left[(1 - \delta) C_t^{1-1/\psi} + \delta (E_t[V_{t+1}^{1-\gamma}])^{1-1/\psi} \right]^{1/(1-1/\psi)}.
\]

Normalized utility obtains if we take the limit as \(\psi \to 1\), divide by \(C_t\), rearrange, and then take the logarithm:

\[
v C_t = \delta \frac{1}{1-\gamma} \log(E_t[e^{(1-\gamma)(\Delta c_{t+1} + v c_{t+1})}]),
\]

Hypothesizing an affine form for \(v C_t\),

\[
v C_t = v_0 + v_c \Delta c_t + v_\pi \pi_t + v_s s_t,
\]

we can substitute to obtain

\[
v_0 + v_c \Delta c_t + v_\pi \pi_t + v_s s_t = \delta \frac{1}{1-\gamma} \log(E_t[e^{(1-\gamma)(\Delta c_{t+1} + v_0 + v_c \Delta c_{t+1} + v_\pi \pi_{t+1} + v_s s_{t+1})}])
\]
and then compute the expectation on the right-hand side:

\[ v_0 + v_c \Delta c_t + v_\pi \pi_t + v_s s_t = \frac{\delta}{1 - \gamma} \left( (1 - \gamma) v_0 + g_0 \left[ (1 - \gamma) (1 + v_c, (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] + g_1 \left[ (1 - \gamma) (1 + v_c, (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] \right) \Delta c_t. \]

Finally, we solve for the coefficients:

\[
\begin{align*}
v_c & : v_c = 0, \\
v_s & : v_s = 0, \\
v_\pi & : v_\pi = v_\pi \delta + \delta (1 - \gamma) v_\pi^2 \sigma^2 / 2 + \frac{\delta}{1 - \gamma} (\varphi (1 - \gamma) - 1), \\
v_0 & : v_0 = v_0 \delta + \mu \delta + v_\pi \delta (1 - \rho) \bar{\pi} - \delta \gamma \sigma^2 / 2,
\end{align*}
\]
where we take the negative root to solve for \( v_\pi \).

In order to derive asset prices, we first solve for the stochastic discount factor:

\[
M_{t+1} = \frac{\delta}{\mathbb{E}_t [e^{(1 - \gamma) (\Delta c_{t+1} + \gamma v_{c_{t+1}})}]} = \delta e^{-\gamma \Delta c_{t+1} + (1 - \gamma) v_{c_{t+1}} + (1 - \gamma) v_{\pi t} + (1 - \gamma) v_{s t}} g_1 \left[ (1 - \gamma) (1 + v_c, (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] \Delta c_t, \pi_{t}, s_t. \]

Since the risk-free rate satisfies \( e^{-r_{f,t}} = \mathbb{E}_t [M_{t+1}] \), we obtain

\[
e^{-r_{f,t}} = \delta e^{(1 - \gamma) v_0 + g_0 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] + g_1 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] \Delta c_t, \pi_{t}, s_t}
\times e^{-(1 - \gamma) v_0 - g_0 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] - g_1 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] \Delta c_t, \pi_{t}, s_t}
\]
and therefore

\[
r_{f,t} = -\log \delta + \mu - \gamma \sigma^2 + \pi_t (\varphi (1 - \gamma) - \varphi (-\gamma)).
\]

Stock returns are therefore given by

\[
r_{d,t+1} = \log \frac{P_{t+1} + D_{t+1}}{P_t} = \log \frac{P_{t+1} / D_{t+1}}{P_t / D_t} + \log \frac{D_{t+1}}{D_t}
\approx k_0 - k_1 (d_{t+1} - p_{t+1}) + d_t - p_t + \Delta d_{t+1}
\]
for some endogenous constants \( k_0 \) and \( k_1 \) to be derived later. We posit an affine form for the logarithm of the D/P ratio:

\[
d_t - p_t = A_0 + A_c \Delta c_t + A_\pi \pi_t + A_s s_t.
\]

The Euler equation for the stock (i.e., the claim asset on \( D_t \)) is \( 1 = \mathbb{E}_t [M_{t+1} e^{r_{d,t+1}}] \). We now plug in \( M_{t+1} \), the log-linearized \( r_{d,t+1} \), and our affine guesses for \( d_t - p_t \) and \( d_{t+1} - p_{t+1} \):

\[
1 = \mathbb{E}_t [\delta e^{-\gamma \Delta c_{t+1} + (1 - \gamma) v_{c_{t+1}} + (1 - \gamma) v_\pi + (1 - \gamma) v_s} - g_0 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] - g_1 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] \Delta c_t, \pi_{t}, s_t}
\times e^{k_0 - k_1 (A_0 + A_c \Delta c_t + A_\pi \pi_t + A_s s_t) + (A_0 + A_c \Delta c_t + A_\pi \pi_t + A_s s_t) + \Delta c_t + \Delta s_t + \Delta s_t}}.
\]

Then we rearrange terms and solve the expectation as follows:

\[
1 = \delta e^{-g_0 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] - g_1 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] \Delta c_t, \pi_{t}, s_t}
\times \mathbb{E}_t \left[ e^{-\gamma \Delta c_{t+1} + (1 - \gamma) v_{c_{t+1}} + (1 - \gamma) v_\pi + (1 - \gamma) v_s} - k_1 (A_0 + A_c \Delta c_t + A_\pi \pi_t + A_s s_t) + \Delta c_t + \Delta s_t + \Delta s_t} \right]
\]

\[
= \delta e^{-g_0 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] - g_1 \left[ (1 - \gamma) (1 - \gamma) v_\pi, (1 - \gamma) v_s) \right] \Delta c_t, \pi_{t}, s_t}
\times e^{g_0 \left[ (1 - \gamma - k_1 A_c (1 - \gamma) v_\pi - k_1 A_\pi (1 - \gamma) v_\pi) \right] - k_1 (A_0 + A_c \Delta c_t + A_\pi \pi_t + (A_s - 1) s_t)
\times e^{g_1 \left[ (1 - \gamma - k_1 A_c (1 - \gamma) v_\pi - k_1 A_\pi (1 - \gamma) v_\pi) \right] - k_1 (A_0 + A_c \Delta c_t + A_\pi \pi_t + (A_s - 1) s_t) + \Delta c_t, \pi_{t}, s_t}}.
\]
Finally, we solve for the coefficients of the D/P ratio and the log linearization constants:

\[
\begin{align*}
    k_0 &= -k_1 \log(k_1) - (1 - k_1) \log(1 - k_1), \\
    \log k_1 &= \log(1 - k_1) - A_0 - A_c \Delta c_t - A_s \Delta \pi_t - A_s \Delta \sigma_t.
\end{align*}
\]

The coefficients satisfy

\[
\begin{align*}
    A_c &= A_c = 0, \\
    A_\pi &= A_\pi = g_1([1 - \gamma, (1 - \gamma)v_\pi, (1 - \gamma)v_s])^{0, 1, 0}', \\
    A_s &= A_s = g_1([1 - \gamma, (1 - \gamma)v_\pi, (1 - \gamma)v_s])^{0, 0, 1}' - 1 \\
    A_0 &= A_0 = -\log \delta - k_1 A_0 - g_0([1 - \gamma - k_1 A_c, (1 - \gamma)v_\pi - k_1 A_\pi, 1 + (1 - \gamma)v_s - k_1 A_s])' \\
    &\quad + g_0([1 - \gamma, (1 - \gamma)v_\pi, (1 - \gamma)v_s])'.
\end{align*}
\]

Hence the equity premium is given by

\[
\begin{align*}
    \log E_t[e^{r_{d,t+1}} - r_{f,t}] &= \log(E_t[e^{r_{d,t+1}}]E_t[M_{t+1}]) \\
    &= \log E_t[e^{r_{d,t+1}}] + \log E_t[M_{t+1}^C] - \log E_t[e^{r_{d,t+1}M_{t+1}^J}] - \text{cov}_t[r_{d,t+1}, m_{t+1}^C] \\
    &= \{1, -k_1 A_\pi, 1 - k_1 A_s\} \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \nu^2 \pi_t & 0 \\ 0 & 0 & \eta^2 \end{bmatrix} \begin{bmatrix} \gamma, (\gamma - 1)v_\pi, (\gamma - 1)v_s \end{bmatrix}' \\
    &\quad + \pi_t[\varphi(1 + \kappa(1 - k_1 A_s)) - \varphi(1 + \kappa(1 - k_1 A_s) - \gamma) + \varphi(-\gamma) - 1],
\end{align*}
\]

where the superscripts $C$ and $J$ denote (respectively) the normal and non-normal components.

The return variance can be expressed as

\[
\begin{align*}
    \text{var}_t[r_{d,t+1}] &= \{1, -k_1 A_\pi, 1 - k_1 A_s\} \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \nu^2 \pi_t & 0 \\ 0 & 0 & \eta^2 \end{bmatrix} \begin{bmatrix} \gamma, (\gamma - 1)v_\pi, (\gamma - 1)v_s \end{bmatrix}' \\
    &\quad + (1 + \kappa(1 - k_1 A_s))^2(\frac{\partial^2}{\partial \varphi \partial \pi_t} \varphi(u)|_{u=0}) \pi_t.
\end{align*}
\]

Finally, the risk-neutral return variance satisfies

\[
\begin{align*}
    \text{var}_t^Q[r_{d,t+1}] &= \{1, -k_1 A_\pi, 1 - k_1 A_s\} \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \nu^2 \pi_t & 0 \\ 0 & 0 & \eta^2 \end{bmatrix} \begin{bmatrix} \gamma, (\gamma - 1)v_\pi, (\gamma - 1)v_s \end{bmatrix}' \\
    &\quad + (1 + \kappa(1 - k_1 A_s))^2(\frac{\partial^2}{\partial \varphi \partial \pi_t} \varphi(u)|_{u=-\gamma}) \pi_t,
\end{align*}
\]

(Drechsler and Yaron, 2011).
Appendix H. Additional Calibration Results

Figure A.IX plots the size distribution of macroeconomic crises. Table A.IV provides historical asset pricing moments and the distribution of their model counterparts for an alternative calibration where using more conservative estimates of disaster size that match the U.S. experience. Figure A.X shows the corresponding predictive slopes.

**Figure A.IX: Size Distribution of Macroeconomic Crises**

*Notes.* The figure plots the empirical density of consumption disasters alongside the density estimated from the shifted gamma distribution. The estimated parameters of the fitted distribution are given in Table VI.
# Table A.IV: Asset Pricing Moments: Conservative Disaster Size

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<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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<th>50%</th>
<th>95%</th>
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<td>Panel A: Unrestricted Model</td>
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<tr>
<td>Average risk-free rate</td>
<td>1.92</td>
<td>0.37</td>
<td>0.72</td>
<td>0.76</td>
<td>0.98</td>
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<td>2.80</td>
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<tr>
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<td>1.42</td>
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<td>0.14</td>
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<tr>
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<td>58.5</td>
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<td>Panel B: Restricted Model (No Realized Disasters)</td>
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<td>71.5</td>
<td>72.8</td>
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</tbody>
</table>

Notes. Panel A reports unconditional moment statistics from S&P 500 real returns and three-month U.S. Treasury real rates as well as percentiles of the same moments from model simulations in our baseline setting with conservative disaster size. Panel B reports the same quantities from a restricted model in which disasters do not realize. Parameters are from Table VI (with the exception of $\alpha = 0.842$). Additional parameters are $\gamma = 9$, $\delta = 99\%$, and $\phi = 2.6$. 
Figure A.X: Model-Implied Predictability with Conservative Disaster Size

Notes. This figure shows predictive slopes from the regression of either future cumulative excess returns (left) or consumption growth (right) on either actual and model-implied macro risk as a function of the horizon. The model accounts for conservative disaster size to match U.S. experience. Model-implied slopes are the median from simulations of both the baseline model (we also report the 5th and 95th percentiles) and the restricted model in which disasters do not realize. Slopes with actual data correspond to point estimates from panel regressions (Tables IV and V).
References


