Disaster Recovery and the Term Structure of Dividend Strips

Michael Hasler† Roberto Marfè‡

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Abstract

Recent empirical findings document downward-sloping term structures of equity return volatility and risk premia. An equilibrium model with rare disasters followed by recoveries helps reconcile theory with empirical observations. Indeed, recoveries outweigh the upward-sloping effect of time-varying disaster intensity and expected growth, generating downward-sloping term structures of dividend growth risk, equity return volatility, and equity risk premia. In addition, the term structure of interest rates is upward-sloping when accounting for recoveries and downward-sloping otherwise. The model quantitatively reconciles high risk premia and a low risk-free rate with the shape of the term structures, which are at odds in other models.

Keywords: Recovery, Time-Varying Rare Disasters, Term Structures of Volatility and Risk Premia

JEL Classification. D51, E21, G12

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†University of Toronto, 105 St. George Street, Toronto, ON, M5S 3E6, Canada, Telephone: +1 (416) 575-2851; michael.hasler@rotman.utoronto.ca; www.utoronto.michael.hasler.ca

‡Collegio Carlo Alberto, Via Real Collegio, 30, 10024 Moncalieri (Torino), Italy, Telephone: +39 (011) 670-5229; roberto.marfe@carloalberto.org; www.carloalberto.roberto.marfe.org
1 Introduction

Recent empirical studies show that the term structures of dividend growth risk, equity return volatility and equity risk premia are downward-sloping (van Binsbergen, Brandt, and Koijen, 2012; van Binsbergen, Hueskes, Koijen, and Vrugt, 2013). These findings are particularly important because they question the validity of the most successful asset-pricing models. In particular, models featuring time-varying disaster risk (Gabaix, 2012; Wachter, 2013) or time-varying expected growth (Bansal and Yaron, 2004) provide convincing foundations for the observed properties of equity return volatility and risk premia, but assume term structures of dividend growth volatility and, in turn, imply term structures of equity return volatility and risk premia that are inconsistent with the data.

In this paper we account for the empirically supported fact that dividends recover after disasters (Gourio, 2008; Nakamura, Steinsson, Barro, and Ursua, 2013). While either natural or man-made disasters affect both physical and, to a lower extent, human capital, it is easy to understand disaster recovery by means of knowledge conservation. Available technology and know-how allow accelerated post-disaster economic growth. Indeed, capital accumulation is easier the second time around because it replicates a known investment pattern. Moreover, disasters induce government spending to stimulate the economic environment and foster competition.\(^2\)

We show that disaster recovery helps explain the observed shape of the term structures of equity return. Indeed, we provide theoretical evidence that recoveries kill the upward-sloping effect of both time-varying disaster risk and time-varying expected growth. The reason is that, in the presence of recoveries, the volatility of dividend growth is larger in the short term than in the long term. In equilibrium, the properties of dividend growth volatility transmit to stock returns and imply empirically consistent downward-sloping term structures of equity return volatility and risk premia.

We base our model on three important properties of the joint dynamics of consumption and dividends for which we provide evidence. First, dividends scaled by consumption are stationary, i.e. consumption and dividends are co-integrated (Lettau and Ludvigson, 2005). Second, the term structure of consumption growth volatility is slightly upward-sloping, whereas that of dividend growth volatility is markedly downward-sloping (Marfè, 2015). This heterogeneity in the timing of fundamental risk is key to understand the shape of the term structures of equity return. Third, conditional on a disaster, dividends drop more and recover faster than consumption (Longstaff and Piazzesi, 2004), validating the theoretical predictions of Gourio (2012).

These stylized facts are closely related to each other. Co-integration implies that con-

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\(^1\)See van Binsbergen and Koijen (2015) for a recent survey on the term structures of equity return.

\(^2\)While there is a debate about the short-run and long-run impacts of disasters on economic growth, developed economies, such as the U.S., seem to be able to mitigate adverse effects and exploit growth opportunities. See Cavallo, Galiani, Noy, and Pantano (2013) and references therein.
sumption and dividends face the same permanent shock, and that the dividend share of consumption is stationary. However, the dividend share of consumption moves negatively with disasters and positively with recoveries. This implies that disasters are (at least partially) transitory shocks and that dividends drop more and recover faster than consumption. The levered exposition of dividends to transitory risk helps explain the gap between the upward-sloping term structure of consumption growth risk and the downward-sloping term structure of dividend growth risk.

We consider a pure-exchange economy (Lucas, 1978) with a representative investor who has Epstein and Zin (1989) preferences. To model the joint dynamics of consumption and dividends, we consider a rare disaster model with time-varying disaster intensity (Wachter, 2013) and expected growth (Bansal and Yaron, 2004). The key feature of our model is that recoveries take place right after the occurrence of a disaster (Gourio, 2008). We argue that, even though disaster intensity risk and long-run risk imply empirically inconsistent term structures of equity return, extending the latter model with plausible recoveries helps explain simultaneously several important properties of dividends, consumption, and asset prices.\(^3\)

We show that the presence of recoveries implies a markedly downward-sloping term structure of dividend growth volatility. The reason is that, in the short term (e.g. 1 day), the dividend faces disaster and expected growth risk, but the horizon is too short to benefit from a significant recovery. In the long term (e.g. 20 years), however, disaster and expected growth risk is still present, but dividends have a significant amount of time to recover. However, the speed of consumption recovery is not high enough to outweigh disaster intensity risk and expected growth risk, both of which imply a large amount of risk in the long term. Therefore, the term structure of consumption growth volatility is slightly upward-sloping.

In equilibrium, equity returns inherit the properties of dividend growth rates. Therefore, the term structure of equity return volatility and risk premia are, as the term structure of dividend growth volatility, downward-sloping in our model. To understand the dynamic patterns of the term structures of equity return over the business cycle, we define bad (resp. good) economic times as states of the world in which the disaster intensity is high (resp. low). Consistent with van Binsbergen, Hueskes, Koijen, and Vrugt (2013), we show that the slopes of the term structures of equity return are pro-cyclical in our model, being smaller in bad times than in good times. The reason is that, in bad times, the disaster intensity is large and is consequently expected to revert back down to its mean in the long term. This implies a significantly larger amount of risk in the short term than in the long term, and therefore steep downward-sloping term structures of equity return. In good times, however, the disaster intensity is small and will eventually revert back up to its long-term mean. That

\(^3\)Consistent with the international evidence documented by Gourio (2008) and Nakamura, Steinsson, Barro, and Ursua (2013), our calibration implies that consumption partially recovers after large drops (by about 50%) in about five years. This is the joint result of long-run growth and after-disaster excessive conditional growth.
is, disaster intensity risk is larger in the long term than in the short term. This dampens the downward-sloping effect of recoveries and implies flatter term structures of equity return. It is worth mentioning that the properties of the term structures of equity return hold even for an elasticity of intertemporal substitution (EIS) smaller than one. The reason is that returns inherit the properties of cash flow growth rates as long as the EIS is larger than some lower bound. This lower bound turns out to be smaller than one for equity—the dividend claim—because dividends have a levered exposition on disaster risk.

In line with empirical findings, the risk-free rate is about 0.6% and equity risk premia are about 6.5%. Interestingly, the model generates relatively large risk premia because, in the presence of recoveries, risk premia decrease with the elasticity of intertemporal substitution (Gourio, 2008). Moreover, the ability of the model to solve the risk-free rate and equity premium puzzle and simultaneously capture the negative slope of the term structures of equity return is robust to the setting of investors’ preferences. In particular, we show that our results hold for an elasticity of intertemporal substitution in the range (1/2,2) and for a coefficient of relative risk aversion in the range (4.4,6.6).

Furthermore, we show that recoveries help explain the observed shape of the term structure of real interest rates. Indeed, the term structure of interest rates is upward-sloping in the presence of recoveries, whereas it is downward-sloping when disasters are permanent. The reason is that the term structure of consumption growth risk is significantly less upward-sloping in the presence of recoveries. Since bonds are used to hedge consumption growth risk, bond yields increase with maturity when there are recoveries and decrease with it when disasters are permanent. Finally, the model implies an inverse relation between the current price-dividend ratio and future excess returns, in line with the empirical findings of Cochrane (2008) and van Binsbergen and Koijen (2010). The predictive power, however, decreases with the forecasting horizon because predictability comes mainly from transitory sources of risk in our model.

Our model builds on the literature about rare disasters (Tsai and Wachter, 2015). Models featuring time-varying risk of disasters provide a theoretical explanation of a number of price patterns (Gabaix, 2012; Wachter, 2013) and have found empirical support (as in Berkman, Jacobsen, and Lee (2011) among others). We complement this literature by pointing out the importance of recovery (Gourio, 2008) and focusing on the otherwise puzzling term structures of both fundamentals growth and equity return.

Since our results rely on the existence of recoveries, our paper is closely related to the findings of Nakamura, Steinsson, Barro, and Ursua (2013). Indeed, Nakamura, Steinsson, Barro, and Ursua (2013) fit a rare disasters model to international consumption data and provide

4 A major critique of long-run risk and rare disasters models is their reliance on a large elasticity of intertemporal substitution (Epstein, Farhi, and Strzalecki, 2014).

5 Note that if disasters are fully permanent, then risk premia increase with the elasticity of intertemporal substitution.
evidence that consumption disasters unfold over a few years, and consumption recovers by about 50% in the following five years. They show that the presence of recoveries significantly decreases equilibrium risk premia on the consumption claim, and reasonably large risk premia can only be obtained if the elasticity of intertemporal substitution is large. In contrast with their study, the dynamics of consumption and dividends are heterogeneous in our model, and we focus on the impact of recoveries on the term structures of consumption growth risk, dividend growth risk, equity return volatility and risk premia, and interest rates. We show that, because dividends are more sensitive to disasters and recoveries than consumption, the shape of each of the five term structures is consistent with empirical findings.

A few recent papers investigate the properties of the term structures of dividend growth risk and equity return. Ai, Croce, Diercks, and Li (2012), Belo, Collin-Dufresne, and Goldstein (2015) and Marfè (2014) focus respectively on investment risk, financial leverage and labor relations: these macroeconomic channels contribute to endogenize the downward-sloping term structure of dividend growth risk and, in turn, of equity return. This paper differs from the aforementioned studies because it focuses on an endowment economy where dividends feature time-varying disaster risk and recover after the occurrence of a disaster. The model calibration provides support to the idea that the disaster-recovery channel has a potentially sizeable quantitative impact on asset price dynamics. Other theoretical studies that help explain the shape of the term structures of equity return focus on non-standard specifications of beliefs formation and preferences. These are proposed by Croce, Lettau, and Ludvigson (2015) and Berrada, Detemple, and Rindisbacher (2013), respectively. In addition, Lettau and Wachter (2007, 2011) show in a partial equilibrium setting that a pricing kernel which enhances short-run risk can help to simultaneously explain the shape of the term structures of equity return and the cross-sectional value premium. Similar to Marfè (2014), our equilibrium framework allows to endogenize the pricing kernel and to provide an economic rationale for the intuition provided in Lettau and Wachter (2007, 2011). As in Longstaff and Piazzesi (2004), we assume that the dividend share of consumption is stationary. While they focus on the relation between the dynamics of corporate cash flows and equity risk premia, our focus is on the impact of recoveries and recursive preferences on the term structures of equity return.

The remainder of the paper is organized as follows: Section 2 provides empirical support to the main assumptions and economic mechanism; Section 3 presents and solves the model; Section 4 describes the term structures of consumption growth risk, dividend growth risk, equity return volatility and risk premia, and interest rates; Section 5 presents the term structures of equity return when disasters unfold over a few years; and Section 6 concludes. Derivations are provided in Appendix A.
Figure 1: Variance ratios of U.S. consumption and dividend growth.

Variance ratios of real U.S. aggregate consumption (non-durable goods and services) growth and real U.S. aggregate dividend growth. The computation of variance ratios accounts for heteroscedasticity and overlapping observations (Campbell, Lo, and MacKinlay, 1997, pp. 48-55). Data are on the sample 1929:2013 at yearly frequency from NIPA tables 1.1.6, 1.1.10 and 1.11.

2 Empirical Support

Gourio (2008) provides evidence that rare disasters are followed by recoveries. This means that, after large drops, GDP and consumption growth feature a conditional mean that is larger than the unconditional one. Such evidence suggests that, in contrast with what is usually considered in the literature, rare disasters should be modelled, at least partially, as transitory shocks instead of permanent shocks. More recently, Nakamura, Steinsson, Barro, and Ursua (2013) provide international evidence that disasters unfold over a few years and then partially recover. Whether disasters have transitory or permanent nature and whether they take place slowly or rapidly are key to the extent of modelling the term structure of risk of macroeconomic fundamentals, i.e. the volatility of growth rates computed over different time horizons or the corresponding variance ratios.

Belo, Collin-Dufresne, and Goldstein (2015) and Marfè (2015, 2014) document that aggregate dividend growth is characterized by a markedly downward-sloping term structure of risk, whereas aggregate consumption growth features a slightly upward-sloping term structure of risk. The corresponding variance ratios are reported in Figure 1. The different shape of the two term structures can be interpreted as follows. Assume that both consumption and dividends are driven by both a permanent and a transitory shock (Lettau and Ludvigson, 2013). These shocks can feature time-varying expected growth, volatility or jump intensity. Variations in the conditional distribution of the permanent shock induce an upward-sloping effect, whereas variations in the conditional distribution of the transitory shock induce a downward-sloping effect. The combination of these two effects determines the shape of the
term structures. As long as consumption and dividends are co-integrated (Lettau and Ludvigson, 2005), they share the same permanent shock but can potentially be exposed differently to the transitory shock. In fact, such a heterogeneity in the exposure to transitory risk explains the difference between the shape of the term structure of consumption growth risk and that of dividend growth risk. Indeed, consumption loads to a lesser extent on the transitory shock than dividends do. Therefore, the upward-sloping effect dominates in the case of consumption and the term structure of consumption growth risk increases with the horizon. In contrast, the downward-sloping effect dominates in the case of dividends and the term structure of dividend growth risk decreases with the horizon. An implication of the aforementioned mechanism is that the dividend share of consumption is stationary and increases with the transitory shock.

In what follows we provide empirical support to the previous interpretation and relate it to the role of disasters and recoveries. Panel A of Table 1 reports summary statistics about real consumption and dividends. Data are from NIPA tables 1.1.6, 1.1.10 and 1.11 and concern the aggregate U.S. economy on the sample 1929-2013 at yearly frequency. Consumption and dividend growth rates share a similar unconditional mean, but the former is smoother than the latter. Consumption growth is positively autocorrelated, whereas dividend growth is negatively autocorrelated. In addition, consumption and dividend growth rates are positively but imperfectly correlated. The excess volatility of dividend growth rates, their negative autocorrelation, and their imperfect correlation with consumption growth rates are three stylized facts consistent with the idea that dividends load more than consumption on a transitory shock. The dividend share is small on average, smooth, relatively persistent, and negatively correlated with the one-period ahead growth rates of consumption and dividends. This is also consistent with the idea that the dividend share is increasing with the transitory component of consumption and dividends.

In Panel B we perform a Johansen test of co-integration in a vector-error-correction model (VECM). Both the Schwarz Bayesian (SBIC) or the Hannan and Quinn (HQIC) information criteria suggest that consumption and dividends are co-integrated. Indeed, we should reject the null hypothesis of no co-integrating equations, and we cannot reject the null hypothesis of one co-integrating equation.

Consistent with the property of co-integration, Panel C shows that the dividend share is stationary. Indeed, the coefficient obtained by regressing the change in the dividend share on its lagged value is negative and significant (the Newey-West t-statistics is $-3.74$). In accord with our interpretation, co-integration implies that the negative slope of dividend growth variance ratios is due to the excessive exposition of dividends to transitory risk.

The upper panel of Figure 2 shows the time series of the logarithm of aggregate dividends and highlights the rare disaster events defined as growth rates smaller than two standard de-

\footnote{This result is robust to the lag and trend specifications, and also holds when using post-war data only.}
Panel A – Summary statistics

<table>
<thead>
<tr>
<th>Consumption growth $\Delta c$</th>
<th>Dividends growth $\Delta d$</th>
<th>Dividend share $D/C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>std</td>
<td>ac(1)</td>
</tr>
<tr>
<td>0.0325</td>
<td>0.0248</td>
<td>0.4956</td>
</tr>
</tbody>
</table>

Correlations: $\Delta c_t, \Delta d_t, \Delta c_t, D/C_{t-1}, \Delta d_t, D/C_{t-1}$

| | | |
| 0.4182 | -0.3867 | -0.3841 |

Panel B – Co-integration

Johansen test (one lag, constant trend)

<table>
<thead>
<tr>
<th>rank</th>
<th>parms</th>
<th>LL</th>
<th>eigenvalue</th>
<th>SBIC</th>
<th>HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>-739.621</td>
<td>22.886</td>
<td>22.845</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>-719.837</td>
<td>0.456</td>
<td>22.469*</td>
<td>22.368*</td>
</tr>
</tbody>
</table>

Panel C – Dividend share stationarity

$\Delta D/C_t = b_0 + b_1 D/C_{t-1} + \epsilon_t$

<table>
<thead>
<tr>
<th>$b_0$</th>
<th>t-stat</th>
<th>$b_1$</th>
<th>t-stat</th>
<th>adj-R$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010***</td>
<td>3.252</td>
<td>-0.215***</td>
<td>-3.736</td>
<td>0.117</td>
</tr>
</tbody>
</table>

Panel D – Dividend share, disasters, and recoveries

$\Delta D/C_t = b_0 + b_1 \text{Dis}_t + b_2 \text{Rec}_t + \epsilon_t$

| | Dis | t-stat | Rec | t-stat | adj-R$^2$ |
| | | | | | |
| (1) | -0.018*** | -4.729 | | | 0.327 |
| (2) | | | 0.016** | 2.281 | 0.159 |
| (3) | -0.017*** | -4.743 | 0.014** | 2.131 | 0.463 |

Table 1: Properties of U.S. consumption and dividends.

Panel A reports the mean, standard deviation, first order autocorrelation, and cross-correlations of real consumption growth rates ($\Delta c$), real dividends growth rates ($\Delta d$), and dividend share of consumption ($D/C$). Panel B reports the Johansen test (one lag, constant trend) for the number of co-integrating equations in a VECM of consumption and dividends. Panel C reports the estimates and Newey-West t-statistics obtained by regressing the change in the dividend share ($\Delta D/C$) on its lagged value. Panel D reports the estimates and Newey-West t-statistics obtained by regressing the change in the dividend share ($\Delta D/C$) on the disaster dummy (Dis) and the recovery dummy (Rec). A disaster-period is defined as years in which growth is smaller than two standard deviation below the average as well as the subsequent negative growth years. A recovery-period is defined as the observation following a disaster period. Statistical significance at the 10%, 5%, and 1% levels is labeled with *, **, and ***, respectively. Data are on the sample 1929-2013 at yearly frequency from NIPA tables 1.1.6, 1.1.10 and 1.11.

viations below the average. Consistent with Gourio (2008) and Nakamura, Steinsson, Barro, and Ursua (2013), we observe that a substantial recovery occurs during the years following
the rare events. The middle panel of Figure 2 shows the time series of the dividend share of consumption. Red markers still represent disasters in dividends. By visual inspection, we observe that the dividend share moves negatively with disasters and positively with recovery. We document this more formally in Panel D of Table 1. We construct a disaster-period dummy (Dis) that is equal to one during years in which growth is smaller than two standard deviations below the average as well as during the subsequent negative growth years. In addition, we construct a recovery dummy (Rec) that is equal to one the year following a disaster. Since the dividend share is stationary, it cannot depend on the permanent component of consumption and dividends. Therefore, the fact that coefficients associated to the dummy variables Dis and Rec are statistically significant provides evidence that the event of a disaster followed by a recovery is (at least partially) transitory. Moreover, the coefficients are respectively negative and positive, consistent with the idea that dividends drop more and recover faster than consumption when a disaster occurs.7

The lower panel of Figure 2 shows the time series of the (standardized) changes in the dividend share and the predicted time series obtained by regressing the changes in the dividend share on the dummy variables Dis and Rec. The joint explanatory power of the dummy variables Dis and Rec is large (adj-R² = 46%), meaning that the disaster/recovery path is a main driver of the dividend share dynamics. In turn, given the stationarity of the dividend share, the disaster/recovery path is a main driver of the transitory component of consumption and dividends. This helps explain why dividend growth is riskier than consumption growth at short horizons, and why the term structures of dividend and consumption growth risk are downward-sloping and upward-sloping, respectively.

Overall, we observe the following stylized facts: i) the timing of dividend growth risk is markedly downward-sloping; ii) the timing of consumption growth risk is slightly upward-sloping; iii) dividend growth is riskier than consumption growth at short horizons; iv) dividends and consumption are co-integrated; v) disasters are followed by recoveries; vi) dividends load more on disaster risk than consumption does, and vii) the dividend share moves with disasters and recoveries.

In Section 3 we present a model that captures the seven stylized facts mentioned above as well as the partial recovery property documented by Nakamura, Steinsson, Barro, and Ursua (2013). In Section 4 we show that the model can simultaneously explain several observed properties of asset prices such as the downward-sloping term structures of equity return volatility and risk premia, the pro-cyclical dynamics of their slopes, the upward-sloping term structure of interest rates, the low risk-free rate, and the high equity risk premia.

7Many asset pricing models, such as Wachter (2013) and Martin (2013) among others, assume that dividends load more on disasters than consumption does. This is consistent with the general equilibrium model of Gourio (2012) in which dividends are endogenous and with the empirical findings of Longstaff and Piazzesi (2004): “while aggregate consumption declined nearly 10% during the early stages of the Great Depression, aggregate corporate earnings were completely obliterated, falling more than 103%.”
Figure 2: U.S. dividends and dividend-share during disasters and recoveries.
The upper and middle panels display the time-series of the logarithm of U.S. aggregate dividends and the standardized U.S. dividend-share of consumption. Red markers denote disasters, i.e. years in which the dividend growth rate is smaller than two standard deviations below the average (and subsequent negative growth years). Green markers denote observations one year ahead of disasters. The lower panel displays the time-series of the standardized changes in the dividend-share and the predicted values obtained from regressing the change in the dividend-share on the disaster dummy and the recovery dummy (see panel D of Table 1). Data are on the sample 1929-2013 at yearly frequency from NIPA tables 1.1.6, 1.1.10 and 1.11.
3 A Model of Rare Disasters and Recoveries

In this section, we first describe the economy and characterize equilibrium asset prices. Then, we calibrate the dynamics of aggregate consumption and dividends to capture the empirical patterns documented in Section 2.

3.1 The Economy

We consider a pure-exchange economy à la Lucas (1978) populated by a representative investor with recursive preferences (Epstein and Zin, 1989). The investor’s utility function is defined by

$$U_t \equiv \left[ (1 - \delta^d t) C_t^{1-\gamma} + \delta^d E_t \left( U_{t+dt}^{1-\gamma} \right)^{\frac{1}{\psi}} \right]^{\frac{\psi}{1-\gamma}},$$

where $C_t$ is consumption, $\delta$ is the subjective discount factor per unit of time, $\gamma$ is the coefficient of risk aversion, $\psi$ is the elasticity of intertemporal substitution (EIS), and $\theta = \frac{1-\gamma}{1-\frac{\gamma}{\psi}}$.

Aggregate consumption, $C_t$, and the dividend paid by equity, $D_t$, are characterized as follows:

$$\log C_t = x_t + z_{1t},$$
$$\log D_t = x_t + z_{2t} + \log d_0, \quad d_0 \in (0, 1),$$

with

$$dx_t = (\mu_t - \frac{1}{2} \sigma_x^2 + \omega z_{1t}) dt + \sigma_x dW_{xt},$$
$$dz_{1t} = - \phi_z z_{1t} dt + \xi_t dN_{zt},$$
$$dz_{2t} = - \phi_z z_{2t} dt + \alpha \xi_t dN_{zt} + \sigma_z dW_{zt},$$
$$d\lambda_t = \phi_{\lambda} (\lambda_t - \lambda_t) dt + \sigma_{\lambda} \sqrt{\lambda_t} dW_{\lambda t},$$
$$d\mu_t = \phi_{\mu} (\mu_t - \mu_t) dt + \sigma_{\mu} dW_{\mu t},$$

where $(W_{xt}, W_{zt}, W_{\lambda t}, W_{\mu t})^T$ is a standard Brownian motion and $N_{zt}$ is a Poisson process with stochastic intensity $\lambda_t$. The jump size $\xi_t$ follows a negative time invariant exponential distribution with parameter $\eta$. That is, the jump size is negative and characterized by the following moment-generating function:

$$\varrho(u) \equiv E_t \left( e^{u \xi_t} \right) = \frac{1}{1 + \frac{u}{\eta}}.$$

The log-consumption and log-dividend processes consist of two components. The first, $x_t$, is the growth rate had there been no disasters and consequently no recoveries either. The aim
of the second component, $z_{it}$, $i \in \{1, 2\}$, is to model disasters and recoveries. Conditional on the occurrence of a disaster ($dN_{zt} = 1$), the log-consumption and log-dividend drop instantaneously by an amount $\xi_t$ and $\alpha \xi_t$, respectively. Following the drop, the process $z_{it}$ reverts back to zero at speed $\phi_z$, and therefore implies a recovery in both consumption and dividends. If the mean-reversion speed $\phi_z$ is equal to zero, there are no recoveries and disasters are permanent. The above dynamics show that if $\alpha = d_0 = 1$ and $\sigma_z = 0$, then consumption is equal to dividends.

Because the permanent shock $x_t$ is a common driver of log consumption and log dividends, consumption and dividends are co-integrated in levels, as observed in the data. In order to obtain an upward-sloping term structure of consumption growth volatility and realistic equity risk premia, we consider time variation in expected growth $\mu_t$ (Bansal and Yaron, 2004). As long as $\phi_z > 0$ a disaster is followed by a recovery, but the recovery is only partial (Nakamura, Steinsson, Barro, and Ursua, 2013) because disasters induce a permanent drop through the expected growth of $x_t$ ($\omega > 0$). In addition, the model embeds a levered exposition of dividends to disasters ($\alpha > 1$). The latter assumption produces the following properties: i) although dividends and consumption are co-integrated, dividend growth is more volatile than consumption growth at short horizons; ii) the term structure of dividend growth risk is downward-sloping because $\alpha > 1$ allows the downward-sloping effect implied by recoveries to outweigh the upward-sloping effect implied by long-run risk and time-varying intensity; and iii) the dividend share $D_t/C_t = d_0 e^{z_{2t} - z_{1t}}$ decreases with disasters and increases with recoveries. The noise parameter $\sigma_z$ is added to the dynamics of dividends because it helps to simultaneously match the consumption growth volatility, the dividend growth volatility, and the dividend-share volatility observed in the data.

### 3.2 Equilibrium

In order to solve for the prices of dividend strips, we follow the methodology documented by Eraker and Shaliastovich (2008), which is based on Campbell and Shiller (1988)’s log-linearization. The first step consists in characterizing the price of the claim to aggregate consumption, the state-price density and therefore the risk-neutral measure. Then, the price at time $t$ of a dividend strip with time-to-maturity $\tau$ is obtained by computing the expected present value under the risk-neutral measure of a dividend $D_{t+\tau}$ paid at time $t + \tau$.

Recursive preferences lead to a non-affine state-price density. Therefore, we make use of

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8Nakamura, Steinsson, Barro, and Ursua (2013) point out that disasters are not necessarily instantaneous. We keep the model simple here and do not account for “slow” disasters because they do not alter the main qualitative result of the paper—recoveries imply downward-sloping term structures of equity return. However, we model “slow” disasters and discuss their implications in Section 5.

9Provided that $\alpha > 1$, this statement holds even though the speed of recovery $\phi_z$ is the same for both log-consumption and log-dividend.

10Even with recursive utility, the risk associated to $\sigma_z$ is not priced. This modeling assumption is intended to make unambiguous the relation between disasters/recoveries and equity risk premia.
the following log-linearization to preserve analytic tractability. The discrete time (continuously compounded) log-return on aggregate wealth $V$ (e.g. the claim on the representative investor’s consumption) can be expressed as

$$\log R_{t+1} = \log \frac{V_{t+1} + C_{t+1}}{V_t} = \log (e^{v_{c,t+1}} + 1) - v_{c,t} + \log \frac{C_{t+1}}{C_t},$$

where $v_{c,t} = \log (V_t/C_t)$. A log-linearization of the first summand around the mean log wealth-consumption ratio leads to

$$\log R_{t+1} \approx k_0 + k_1 v_{c,t+1} - v_{c,t} + \log \frac{C_{t+1}}{C_t},$$

where the endogenous constants $k_0$ and $k_1$ satisfy

$$k_0 = -\log \left( (1 - k_1)^{1-k_1} k_1^{k_1} \right) \quad k_1 = e^{E(v_{c,t})} / \left( 1 + e^{E(v_{c,t})} \right).$$

In a Gaussian environment, Campbell, Lo, and MacKinlay (1997) and Bansal, Kiku, and Yaron (2012) document the high accuracy of such a log-linearization, which we assume exact hereafter.\footnote{In Appendix A.5 we show that the log-linearization remains accurate when the model accounts for disasters (i.e. jumps in consumption). It is worth noting that we perform a log-linearization of the wealth return dynamics only. Indeed, we do not need to perform a second log-linearization when computing either dividend strip prices or the price of equity. Therefore, the log-linearization affects prices only through the intertemporal component of the state-price density, which is a function of wealth under recursive utility. Moreover, we show in Section 4 that dividend strip returns inherit the term-structure properties of dividend growth rates. That is, our results are mostly implied by the cash flow channel as opposed to the discount rate channel. In turn, the quantitative impact of the log-linearization on our results is likely to be negligible.}

We follow Eraker and Shaliastovich (2008) and consider the continuous time counterpart defined in the following way:

$$d \log R_t = k_0 dt + k_1 dv_{c,t} - (1 - k_1) v_{c,t} dt + d \log C_t.$$ 

Recursive preferences lead to the following Euler equation, which enables us to characterize the state-price density, $M_t$, used to price any asset in the economy:

$$\mathbb{E}_t \left[ \exp \left( \log \frac{M_{t+\tau}}{M_t} + \int_t^{t+\tau} d \log R_s \right) \right] = 1.$$ 

The state-price density satisfies

$$d \log M_t = \theta \log \delta dt - \frac{\theta}{\psi} d \log C_t - (1 - \theta) d \log R_t.$$ 

To solve for the return on aggregate wealth and, in turn, on the state-price density, one has to conjecture that $v_{c,t}$ is affine in the vector of state variables $Y_t = (x_t, z_{1t}, z_{2t}, \lambda_t, \mu_t)\top$. Then, the Euler equation is used to solve for the coefficients. The wealth of the representative
The investor is characterized in Proposition 1 below.

**Proposition 1.** The investor’s wealth, \( V_t \), satisfies

\[
v_{c,t} = \log \frac{V_t}{C_t} = A_c + B_c^T Y_t,
\]

where “\( \top \)” is the transpose operator and \( Y_t = (x_t, z_{1t}, z_{2t}, \lambda_t, \mu_t)^\top \) is the vector of state variables. The state variables belong to the affine class and their dynamics can be written as:

\[
dY_t = \mu(Y_t)dt + \Sigma(Y_t)dW_t + J_t \cdot dN_t
\]

where \( \mu(Y_t) = M + KY_t \)

\[
\Sigma(Y_t)\Sigma(Y_t)^\top = h + \sum_{i=1}^{5} H^i Y^i_t
\]

\[
l(Y_t) = l_0 + l_1 Y_t,
\]

The coefficients \( A_c \in \mathbb{R} \) and \( B_c \in \mathbb{R}^5 \) solve a system of equations provided in Appendix A.1.

**Proof.** See Appendix A.1.

The wealth-consumption ratio is stationary in our model and consequently independent of \( x_t \) \((B_{c,1} = 0)\). Moreover, the wealth-consumption ratio decreases (resp. increases) with the intensity \( \lambda_t \) when the elasticity of intertemporal substitution is larger (resp. smaller) than one. The reason is that an increase in the intensity increases the likelihood that negative jumps in consumption will take place in the future. This has two opposite effects on the wealth-consumption ratio. First, the possibility of more frequent downward jumps implies lower expected future consumption. Since the wealth-consumption ratio is the (scaled) expectation of discounted future consumption, an increase in the intensity tends to decrease the wealth-consumption ratio through a substitution effect. Second, lower expected future consumption pushes the investor to save more today for future consumption purposes. This income effect tends to increase the wealth-consumption ratio. The substitution effect dominates when the elasticity of intertemporal substitution is larger than one, whereas the income effect is the strongest when the elasticity is smaller than one.

For a similar reason, the wealth-consumption ratio decreases (resp. increases) with the jump process \( z_{1t} \) when the EIS is larger (resp. smaller) than one. Indeed, a drop in \( z_{1t} \), today, indicates that future consumption growth will be large because a recovery will take
place. Since the substitution effect dominates when the EIS is larger than one, a drop in $z_{1t}$ implies an increase in the wealth-consumption ratio. Conversely, a drop in $z_{1t}$ implies a decrease in the wealth-consumption ratio when the EIS is smaller than one because, in this case, the income effect dominates.

The risk-free rate $r_t$, the market price of continuous risk $\Lambda_t^c$, and the market price of jump risk $\Lambda_t^d$ are defined in Proposition 2.

**Proposition 2.** The dynamics of the state-price density, $M_t$, are written

$$
\frac{dM_t}{M_t} = -r_t dt - \Lambda_t^c dW_t - \sum_{i=1}^5 \left( \Lambda_t^{di} dN_i^t - \mathbb{E}_t \left( \Lambda_t^{di} \right) l^i(Y_t) dt \right),
$$

where the risk-free rate $r_t$, the market price of continuous risk $\Lambda_t^c$, and the market price of jump risk $\Lambda_t^d$ satisfy

$$
\begin{align*}
r_t &= \Phi_0 + \Phi_1^T Y_t, \\
\Lambda_t^c &= \Sigma(Y_t)^T \Omega, \\
\Lambda_t^{di} &= 1 - e^{-\Omega J_t}, \quad \forall i.
\end{align*}
$$

The coefficient $\Phi_0 \in \mathbb{R}$ and $\Phi_1 \in \mathbb{R}^5$ solve a system of equations provided in Appendix A.2.

**Proof.** See Appendix A.2.

The risk-free rate is stationary and therefore independent of $x_t$. It is, however, a decreasing linear function of the intensity $\lambda_t$, irrespective of the value assigned to the EIS. The reason is that an increase in the disaster intensity yields an increase in consumption growth risk, which the investor is willing to hedge with risk-free investments. This willingness to increase risk-free holdings implies an increase in the price of the risk-free asset and thus a decrease in the risk-free rate. Such an effect increases in magnitude with relative risk aversion. The decreasing linear relation between the risk-free rate and the disaster process $z_{1t}$ is understood as follows. A disaster, i.e. a drop in $z_{1t}$ today, will be followed by a recovery and therefore implies high expected future consumption growth. This decreases the investor’s willingness to hedge using the risk-free asset, and hence increases the risk-free rate.

The first component of the market price of continuous risk $\Lambda_t^c$ is constant and equal to $\gamma \sigma_x$ (Lucas, 1978); it rewards the investor for bearing the constant dividend growth risk. The second component of $\Lambda_t^c$ is associated with the jump intensity. This term is stochastic because the jump intensity follows a square-root process (see Equation (1)). The last component of $\Lambda_t^c$ is associated with time-variation in expected growth. The latter is positive (resp. negative) under preference for early (resp. late) resolution of uncertainty (Bansal and Yaron, 2004). Moreover, the investor gets a reward $\Lambda_t^d$ for bearing the risk of a jump in $z_{1t}$. This term is constant because the jump size $\xi_t$ features an i.i.d. distribution.
Proposition 3 characterizes the prices of consumption strips, dividend strips, and non-defaultable bonds with time to maturity $\tau$.

**Proposition 3.** Prices of consumption strips $S^{\text{con}}_t(\tau)$, dividend strips $S^{\text{div}}_t(\tau)$, and non-defaultable bonds $B_t(\tau)$ with time to maturity $\tau$ satisfy:

$$S^{\text{con}}_t(\tau) = \mathbb{E}_t^\mathbb{Q} \left( e^{\int_t^{t+\tau} r_s ds} C_t + \tau \right),$$

$$S^{\text{div}}_t(\tau) = \mathbb{E}_t^\mathbb{Q} \left( e^{\int_t^{t+\tau} r_s ds} D_t + \tau \right),$$

$$B_t(\tau) = \mathbb{E}_t^\mathbb{Q} \left( e^{\int_t^{t+\tau} r_s ds} \right) = e^{a_B(\tau) + b_B(\tau)^\top Y_t},$$

where $\mathbb{Q}$ is the risk-neutral measure defined by Proposition 2. The deterministic functions $a_{\text{con}}(\cdot), a_{\text{div}}(\cdot), a_B(\cdot) \in \mathbb{R}$ and $b_{\text{con}}(\cdot), b_{\text{div}}(\cdot), b_B(\cdot) \in \mathbb{R}^5$ solve a system of ordinary differential equations provided in Appendix A.3.

**Proof.** See Appendix A.3.

The valuation ratios $S^{\text{con}}_t(\tau)/C$, $S^{\text{div}}_t(\tau)/D$, and $B_t(\tau)$ do not depend on $x$ and, hence, are stationary. Similar to Proposition 4, an application of Itô’s lemma to the dividend strip price, $S^{\text{div}}_t(\tau)$, allows us to derive the term structures of both equity return volatility, $||\sigma^{\text{div}}_t(\tau)||$, and risk premia $R P^{\text{div}}_t(\tau)$.

The price of equity (i.e. the claim on dividends) can be either computed by numerical integration of the dividend strip prices over the time horizon

$$P_t = \int_0^\infty S^{\text{div}}_t(\tau) d\tau,$$

or approximated through a log-linearization of the return process. In that case, the price satisfies the following exponential affine form: $P_t \approx D_t e^{A d + B_d^\top Y_t}$. The latter expression is particularly convenient when pricing other assets such as options (see Eraker and Shaliastovich (2008)).

The conditional return volatility and risk premia of a strip with time to maturity $\tau$ is provided in Proposition 4.

**Proposition 4.** The time $t$ return variance of a strip with time to maturity $\tau$ is written

$$||\sigma^{CF}_t(\tau)||^2 = ||\sigma^{CF,c}_t(\tau)||^2 + ||\sigma^{CF,d}_t(\tau)||^2,$$

where $CF \in \{\text{con}, \text{div}\}$ determines either the consumption strip or the dividend strip, and $||\sigma^{CF,c}_t(\tau)||^2$ and $||\sigma^{CF,d}_t(\tau)||^2$ are the return variances implied by the Brownian motion $W_t$. 

16
and the jump process $N_t$, respectively. The vectors $\sigma_{t}^{CF,c}(\tau)$ and $\sigma_{t}^{CF,d}(\tau)$ satisfy

$$
\sigma_{t}^{CF,c}(\tau) = \frac{1}{S_{t}^{CF}(\tau)} \left( \frac{\partial}{\partial Y} S_{t}^{CF}(\tau) \right)^{\top} \Sigma(Y_t)
$$

$$
\sigma_{t}^{CF,d}(\tau) = \sqrt{l(Y_t)^i E \left[ (e^{b_i^{CF}(\tau)Y_t^{i}} - 1)^2 \right]}
= \sqrt{l(Y_t)^i (\varphi(2b_i^{CF}(\tau)) - 2\varphi(b_i^{CF}(\tau)) + 1)}.
$$

The time $t$ risk premia on a strip with time to maturity $\tau$ are written

$$
RP_{t}^{CF}(\tau) = RP_{t}^{CF,c}(\tau) + RP_{t}^{CF,d}(\tau),
$$

where $RP_{t}^{CF,c}(\tau)$ and $RP_{t}^{CF,d}(\tau)$ are the premia for bearing Brownian and jump risks, respectively. These risk premia satisfy

$$
RP_{t}^{CF,c}(\tau) = \sigma_{t}^{CF,c}(\tau) \Lambda_{t}^{c}
$$

$$
RP_{t}^{CF,d}(\tau) = \sum_{i=1}^{5} l(Y_t)^i E \left[ (e^{b_i^{CF}(\tau)Y_t^{i}} - 1) \Lambda_{t}^{d} \right]
= \sum_{i=1}^{5} l(Y_t)^i (\varphi(b_i^{CF}(\tau)) - \varphi(b_i^{CF}(\tau) - \Omega^i) + \varphi(-\Omega^i) - 1).
$$

**Proof.** Application of Itô’s lemma and Girsanov’s theorem.

In Section 4, we show that the level and shape of the term structures of equity return volatility and risk premia are inherited by the level and shape of the term structure of dividend growth volatility when the EIS is large enough.

### 3.3 Calibration

The model is calibrated as follows. First, we fit the probability of disasters, the average size of disasters, the average size of recoveries, and the length of recoveries to the values estimated by Gourio (2008), Barro and Ursua (2008), and Nakamura, Steinsson, Barro, and Ursua (2013). The empirical probability of disasters is in the range $(0.028, 0.036)$, whereas the empirical size of disasters is in the range $(0.22, 0.32)$. To be conservative, we pick the lower bounds and therefore set $\lambda = 0.028$ and $\eta = 4.02$. The parameters $\omega = 0.108$ and $\phi_z = 0.302$ imply a 50% recovery from disasters in approximately five years. That is, 50% of the disaster size is permanent. Second, the remaining parameters are such that the yearly mean of consumption and dividend growth, the yearly volatility of consumption and dividend growth, the long-term (10-year) variance ratio of consumption and dividend growth, and the mean and volatility of the dividend share are as close as possible to their empirical counterparts. Table 2 shows the empirical and theoretical moments of consumption and dividends, and Table 3 provides the corresponding parameter values. It is worth noting that the size and
<table>
<thead>
<tr>
<th>Moment</th>
<th>Empirical Value</th>
<th>Theoretical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption disasters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of occurrence</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>Size of the drop</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Size of recovery</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Length of recovery (years)</td>
<td>$\sim 5$</td>
<td>$\sim 5$</td>
</tr>
<tr>
<td><strong>Consumption growth:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.025</td>
<td>0.04</td>
</tr>
<tr>
<td>10-year variance ratio</td>
<td>1.01</td>
<td>1.06</td>
</tr>
<tr>
<td><strong>Dividend growth:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>10-year variance ratio</td>
<td>0.37</td>
<td>0.29</td>
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<tr>
<td><strong>Dividend share:</strong></td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.048</td>
<td>0.05</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.014</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 2: Empirical and theoretical moments of consumption and dividends. Consumption and dividend data are on the sample 1929-2013 at yearly frequency from NIPA tables 1.1.6 and 1.10.

frequency of disasters imply substantial consumption growth volatility. Therefore, we set $\sigma_x = 0$ such that consumption growth is smooth and varies only with the expected growth rate, disasters, and recoveries.

4 Results

In this section, we first discuss the properties of the term structure of dividend growth risk because it is the main determinant of the shape of the term structures of equity return volatility and risk premia. Then, we show how preferences and economic conditions impact the term structures of equity return volatility, equity risk premia, and interest rates. The model can simultaneously explain the shape of the term structures of equity return and interest rates, while providing a solution to the equity premium and risk-free rate puzzles. In addition, we show that the price-dividend ratio predicts future excess returns in the model, consistent with empirical findings.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Permanent shock:</td>
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<td></td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td>$\sigma_x$</td>
<td>0</td>
</tr>
<tr>
<td>Long-run expected growth</td>
<td>$\bar{\mu}$</td>
<td>0.033</td>
</tr>
<tr>
<td>Volatility of expected growth</td>
<td>$\sigma_\mu$</td>
<td>0.01</td>
</tr>
<tr>
<td>Speed of reversion of expected growth</td>
<td>$\phi_\mu$</td>
<td>0.2</td>
</tr>
<tr>
<td>Disasters:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed of reversion of jump intensity</td>
<td>$\phi_\lambda$</td>
<td>0.2</td>
</tr>
<tr>
<td>Long-term jump intensity</td>
<td>$\lambda$</td>
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<tr>
<td>Volatility of jump intensity</td>
<td>$\sigma_\lambda$</td>
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</tr>
<tr>
<td>Jump size parameter</td>
<td>$\eta$</td>
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</tr>
<tr>
<td>Permanent disaster parameter</td>
<td>$\omega$</td>
<td>0.108</td>
</tr>
<tr>
<td>Speed of recovery</td>
<td>$\phi_z$</td>
<td>0.302</td>
</tr>
<tr>
<td>Dividends:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility of dividend growth</td>
<td>$\sigma_z$</td>
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</tr>
<tr>
<td>Leverage parameter</td>
<td>$\alpha$</td>
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<td>Steady state dividend share</td>
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</tr>
<tr>
<td>Preferences:</td>
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<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>5.5</td>
</tr>
<tr>
<td>Elasticity of intertemporal substitution (EIS)</td>
<td>$\psi$</td>
<td>0.8</td>
</tr>
<tr>
<td>Time discount factor</td>
<td>$\delta$</td>
<td>0.963</td>
</tr>
</tbody>
</table>

Table 3: Calibration.

4.1 Term Structures of Consumption and Dividend Growth Risk

The conditional term structures of cash flow growth volatility and variance ratios are characterized in Proposition 5 below.

**Proposition 5.** The time $t$ cash flow growth volatility at a $\tau$-year horizon, $vol^{CF}(t, \tau)$, is written

$$vol^{CF}(t, \tau) = \frac{1}{\tau} \log \left( \frac{MGF^{CF}(t, \tau; 2)}{MGF^{CF}(t, \tau; 1)^2} \right),$$

where $CF \in \{\text{con}, \text{div}\}$ determines whether the cash flow is the consumption $C$ or the dividend $D$, and the conditional moment-generating function $MGF^{CF}(., ., .)$ satisfies

$$MGF^{CF}(t, \tau; u) = \mathbb{E}_t \left( F^{u}_t \right) = e^{\bar{a}_{CF}(\tau; u) + \bar{b}_{CF}(\tau; u)^\top Y_t},$$

where $F = C$ if $CF = \text{con}$ and $F = D$ if $CF = \text{div}$. The deterministic functions $\bar{a}_{CF}(., .) \in \mathbb{R}$.
and $\bar{b}_{CF}(\cdot, \cdot) \in \mathbb{R}^5$ solve a system of ordinary differential equations provided in Appendix A.4. The time $t$ cash flow growth variance ratio at a $\tau$-year horizon, $VR^{CF}(t, \tau)$, is defined as follows:

$$VR^{CF}(t, \tau) = \frac{vol^{CF}(t, \tau)^2}{vol^{CF}(t, 1)^2}.$$ 

**Proof.** See Appendix A.4.

Notice that $vol^{CF}(t, \tau)$ captures both the continuous and the discontinuous expected variation in the cash flow growth. Moreover, the first component of $\bar{b}_{CF}(\tau; u)$ is simply $u$, such that $vol^{CF}(t, \tau)$ and $VR^{CF}(t, \tau)$ do not depend on $x_t$.

Figure 3 displays the term structures of dividend growth and consumption growth volatility (on the left) and the corresponding variance ratios (on the right). In line with empirical findings (see Figure 1), the model implies strongly downward-sloping and slightly upward-sloping term structures of dividend growth volatility and consumption growth volatility, respectively. That is, accounting for recoveries helps explain the observed shape of the term structures of dividend growth and consumption growth risk.\textsuperscript{12} The reason is that the three main components of cash flow growth risk are the disaster risk, the intensity risk, and the expected growth risk.\textsuperscript{13} Time variation in jump intensity, $\lambda_t$, and expected growth, $\mu_t$, implies an upward-sloping effect on the term structure because the longer the horizon is, the higher the uncertainty concerning the frequency of disasters and the propensity to grow becomes.\textsuperscript{14} Consider now the role of $z_{1t}$ and $z_{2t}$. Over short horizons (e.g. 1 day), recoveries do not influence disaster risk. Over long horizons (e.g. 20 years), however, recoveries dampen the disaster risk that would have prevailed, had there been no recoveries ($\phi_z = 0$). Therefore, recovery from disasters implies a downward-sloping effect on the term structure. Overall, the downward-sloping effect of recovery dominates the upward-sloping effect of time-varying intensity and expected growth when cash flows are sufficiently exposed to disasters and recoveries, i.e. when $\alpha$ is sufficiently large. This generates a downward-sloping term structure of dividend growth risk and a slightly upward-sloping term structure of consumption growth risk.

In what follows, we first analyze the model predictions concerning the term structures of equity return and interest rates. Second, we show how the model reconciles a solution to the

\textsuperscript{12}Beeler and Campbell (2012), Belo, Collin-Dufresne, and Goldstein (2015), and Marfè (2014) provide empirical evidence of a decreasing term structure of dividend growth variance ratios.

\textsuperscript{13}Note that we do not discuss the risk implied by the Brownian motion $W_{xt}$ because it has no effect on the slope of the term structure (i.e. this Brownian risk is proportional to the time horizon).

\textsuperscript{14}In the absence of recoveries, both time-varying disaster intensity and time-varying expected growth imply strongly upward-sloping term structures of cash flow growth risk and, in turn, of equity return volatility and risk premia (van Binsbergen, Brandt, and Koijen, 2012).
Figure 3: Term structure of volatility and variance ratio for consumption and dividend growth.
The left panel depicts the volatility of consumption and dividend growth for horizons ranging from 0 to 20 years. The right panel depicts the corresponding variance ratios. That is, the \( \tau \)-year variance relative to the 1-year variance. State variables are \( x_t = z_{1t} = z_{2t} = 0, \lambda_t = \lambda \), and \( \mu_t = \bar{\mu} \) and the calibration is provided in Table 3.

...risk-free rate and equity premium puzzles with an adequate description of the term structures of consumption growth risk, dividend growth risk, equity return, and interest rates. Third, we show that the model implies time series predictability patterns that are in line with the data.

4.2 Term Structures of Equity Return and Interest Rates

Figure 4 displays the term structures of return volatility (left) and risk premia (right) for the dividend and consumption strips. When the elasticity of intertemporal substitution is lower than one, the income effect dominates the substitution effect and the term structures associated with the strip on consumption are slightly upward-sloping. However, when dividends load more than consumption on disaster risk, even a moderate substitution effect (e.g. \( \psi < 1 \)) is sufficient to generate downward-sloping term structures of dividend strips. Indeed, equity compensation is high in the short term because dividends are highly sensitive to disaster risk (\( \alpha > 1 \)), and disaster risk concentrates at short horizons in the presence of recoveries (\( \phi_z > 0 \)). The larger \( \alpha \) is, the smaller the elasticity of intertemporal substitution needed to obtain the observed slopes of the term structures of equity return.

Figure 5 describes the relation between the elasticity of intertemporal substitution and the slopes of the term structures. We observe two noteworthy results. First, equity return risk equals dividend growth risk at very short horizons (\( \tau \to 0 \)) and inherits the negative slope of dividend growth risk. However, the larger the elasticity of intertemporal substitution, the steeper the negative slope of equity return risk becomes. Then, a moderate elasticity of
intertemporal substitution (e.g. $\psi = 0.8$) helps to dampen the excess volatility of dividend growth relative to equity return at long horizons—a counter-factual implication of several asset-pricing models (see Beeler and Campbell, 2012).

Second, the slope of equity risk premia is negative and the level of risk premia is decreasing with the elasticity of intertemporal substitution. Dividend strip risk premia lie between 6% and 8% when $\psi = 0.8$, whereas they lie between 3% and 4% when $\psi = 1.5$. As recognized by Gourio (2008), required premia for bearing disaster risk are decreasing with $\psi$ in the presence of recovery. The reason is twofold. First, recoveries imply a decreasing term structure of dividend growth risk, meaning that risk is concentrated in the short term. Second, the investor requires a low (resp. high) premium for bearing short-term risk when he has a preference for early (resp. late) resolution of uncertainty. Since an increase in the elasticity of intertemporal substitution implies a preference for earlier resolution of uncertainty, the relation between risk premia and the parameter $\psi$ is negative. The opposite holds when the term structure of dividend growth risk is upward-sloping, i.e. when there is no recovery. In this case, risk premia increase with $\psi$ because the larger the elasticity of intertemporal substitution is, the earlier the investor is willing to resolve uncertainty, and therefore the larger the required compensation for bearing long-term risk.

This reasoning explains why previous disaster models (Wachter, 2013), as well as long-run risk models (Bansal and Yaron, 2004), have difficulty reconciling downward-sloping term structures with high equity risk premia. Indeed, both classes of models assume cash flow dynamics that imply an upward-sloping term structure of dividend growth risk. Therefore, these models can either generate high risk premia and upward-sloping term structures of
equity return if the agent has a preference for early resolution of uncertainty ($\gamma > 1/\psi$) or low risk premia and downward-sloping term structures of equity return if the agent has a preference for late resolution of uncertainty ($\gamma < 1/\psi$).

Our setup mitigates this tension by improving the timing of consumption and dividend growth risk. Even for relatively low preference parameters ($\gamma = 5.5$, $\psi = 0.8$), the (partially) transitory nature of disasters ($\phi_z > 0$) and the larger sensitivity of dividends relative to consumption on disaster risk ($\alpha > 1$) are sufficient to generate high compensations and downward-sloping term structures of equity return.

As in Wachter (2013), we assume that the disaster intensity is a proxy for economic conditions. That is, good, normal, and bad times correspond to a low, moderate, and high disaster intensity, respectively. Figure 6 shows that the term structures of equity return are downward-sloping and flatten when economic conditions improve. As mentioned earlier, the fact that dividends are expected to recover in the long term implies less risk in the long term than in the short term and therefore downward-sloping term structures of equity return. When economic conditions are good, the disaster intensity is expected to revert back up to its long-term mean. This increases the dividend growth risk in the long term and therefore dampens the downward-sloping effect implied by recoveries. As shown by van Binsbergen, Hueskes, Koijen, and Vrugt (2013), the pro-cyclical dynamics of the slope of the term structure of equity risk premia finds empirical support.

Figure 7 depicts the term structures of real interest rates implied by the model. The term structure of interest rates is downward-sloping when there are no recoveries and upward-sloping when there are recoveries. The reason is that the term structure of consumption

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**Figure 5: Term structure of equity return volatility and risk premia vs. elasticity of intertemporal substitution.**

The left panel depicts the term structure of equity return volatility for horizons ranging from 0 to 20 years. The right panel depicts the term structure of equity risk premia. Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$ and the calibration is provided in Table 3.
Figure 6: Term structure of equity return volatility and risk premia vs. economic conditions.

The left panel depicts the term structure of equity return volatility for horizons ranging from 0 to 20 years. The right panel depicts the term structure of equity risk premia. Good, normal, and bad times correspond to a jump intensity $\lambda_t = \bar{\lambda} - 0.0275$, $\lambda_t = \bar{\lambda}$, and $\lambda_t = \bar{\lambda} + 0.0275$, respectively. Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$ and the calibration is provided in Table 3.

growth risk is significantly less upward-sloping in the latter case. Since bonds are hedging instruments, bond prices are positively related to consumption growth risk, and bond yields are inversely related to it. Therefore, bond yields decrease with maturity when there are no recoveries and increase with it otherwise. This shows that recoveries not only help understand the observed shape of the term structures of equity return, but also that of the term structure of interest rates.

The term structure of real interest rates flattens as the EIS increases (bottom left panel of Figure 7). The reason is that bond prices are defined as the expectation under the risk-neutral measure of a $1 future cash flow discounted at the risk-free rate (Equation (2)). Therefore, the term structure is flat if the risk-free rate is constant (no volatility) and steeper if the risk-free rate is volatile. Because the volatility of the risk-free rate mainly depends on the product of the volatility of the expected growth rate and the inverse of the EIS, it decreases with the EIS (Bansal and Yaron, 2004). That is, an increase in EIS implies a decrease in the volatility of the risk-free rate, and therefore a flatter term structure of interest rates.

Consistent with the findings of Buraschi and Jiltsov (2007), the slope of the term structure of real interest rates is counter-cyclical in our model (bottom right panel of Figure 7). In good times, the intensity of disaster $\lambda_t$ is low, which implies low consumption growth risk in the short term and therefore a small demand for short-term bonds. That is, short-term bonds are cheap and their yields high. In the longer term, investors expect the intensity to revert back up to its long-term mean, which increases long-term consumption growth risk and decreases bond yields. In bad times the opposite holds. Indeed, consumption growth risk
is high in the short term because the intensity of disaster is high. Consequently, short-term bonds are expensive and their yields low. Since investors expect the intensity of disaster to revert back down to its long-term mean, long-term consumption growth risk is low and therefore long-term bond yields are high.

4.3 Equity Risk Premia, Risk-free Rate, and Return Predictability

To complete the analysis, we look at the model-implied levels of the risk-free rate and equity risk premia. Table 4 shows how these two variables depend on the pairs of preference parameters $\gamma$ and $\psi$. Equity premia increase with relative risk aversion and decrease with
the elasticity of intertemporal substitution. As noted above, the latter result comes from the fact that the main source of risk premia is disaster risk, which is (partially) transitory in the presence of recovery ($\phi_z > 0$). We observe that the pair $\gamma = 5.5$ and $\psi = 0.8$ leads to sizeable and realistic equity risk premia (about 6.3%) and a low risk-free rate (about 0.9%). Therefore, the model reconciles the risk-free rate and equity premium puzzle with the recent empirical evidence concerning the term structures of equity (van Binsbergen, Brandt, and Koijen, 2012). The levered exposition of dividends to disaster risk is a property of the data (see Section 2), which helps obtain downward-sloping term structures of equity return for values of EIS smaller than one. Because equity premia are decreasing with the EIS in the presence of recovery, dividend leverage ($\alpha > 1$) helps match, both qualitatively and quantitatively, the observed term structures of equity return and the average level of equity premia.

There is a debate about the values of the EIS used in theoretical models and those estimated using real data (see Hall (1988) and Epstein, Farhi, and Strzalecki (2014)). Indeed, theoretical models are usually very sensitive to the EIS and generate realistic price patterns only under specific restrictions on the EIS (e.g. Bansal and Yaron, 2004). In what follows, we want to verify whether our results are robust to the choice of preference parameters, and in particular to the choice of the EIS. Figure 8 shows the pairs of relative risk aversion and time discount factor ($\gamma, \delta$), which allow to match the empirical levels of the risk-free rate and equity risk premia for values of the EIS in the range $(1/2, 2)$. We require a risk-free rate of 0.6% and equity premia of 6.5% (Constantinides and Ghosh, 2011).

Several results are noteworthy. First, the model can quantitatively solve the risk-free rate and equity premium puzzle for any value of EIS within this range. Second, the pairs ($\gamma, \delta$) required by the model belong to the ranges $\delta \in (0.95, 0.98)$ and $\gamma \in (4.4, 6.6)$, which are

---

**Table 4: Risk-free rate and equity premia vs. preference parameters: comparative statics.**

Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$ and the calibration is provided in Table 3.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\psi = 0.8$</th>
<th>$\psi = 1$</th>
<th>$\psi = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r</td>
<td>RP</td>
<td>r</td>
</tr>
<tr>
<td>$\gamma = 4$</td>
<td>4.79%</td>
<td>2.42%</td>
<td>4.68%</td>
</tr>
<tr>
<td>$\gamma = 5.5$</td>
<td>0.87%</td>
<td>6.30%</td>
<td>2.09%</td>
</tr>
<tr>
<td>$\gamma = 7$</td>
<td>-28.24%</td>
<td>34.35%</td>
<td>-10.73%</td>
</tr>
</tbody>
</table>

Note that these parameters imply a relatively low equity volatility (about 6%). The level of volatility can be increased by decreasing the EIS, at the expense of generating a too low risk-free rate and too high risk premia.
considered to be plausible in the existing literature. Since equity premia are decreasing with the EIS when there are recoveries, an increase in both the EIS and relative risk aversion is needed to match observed equity premia. Third, for each value of the EIS within this range, the pair \((\gamma, \delta)\) required to match the observed risk-free rate and equity premia also implies downward-sloping term structures of equity return. That is, the speed of recovery from disasters implies a shift of equity risk towards the short term that is robust to a wide range of preference parameters. This provides further support to the idea that recovery from disasters in macroeconomic fundamentals is a key driver of asset prices.

In what follows, we investigate the predictability of excess stock returns implied by our model. To do so, we regress long-horizon excess returns on the log price-dividend ratio. The regression is performed on returns measured over horizons ranging from 1 to 7 years. In each of our one thousand simulations, the price-dividend ratio and excess returns are simulated at the monthly frequency over a 60-year horizon. The choice of the frequency and horizon matches that of most of the empirical literature on predictive regressions.

Panel A of Table 5 shows that the model reproduces the negative and statistically significant intertemporal relationship between the current valuation ratios and future excess returns that we observe in actual data (Cochrane, 2008; Lettau and van Nieuwerburgh, 2008; van Binsbergen and Koijen, 2010). In contrast with empirical evidence, however, the forecasting power of the price-dividend ratio decreases with the horizon\(^{16}\) because most of the predictability comes out from transitory risk in fundamentals. Therefore, recoveries from disasters help

\(^{16}\) Lettau and Wachter (2007) suggest some caution when discussing the empirical properties of R\(^2\) statistics because they are quite unstable over different sample periods.
Panel A – Model simulations with realized disasters

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficient</td>
<td>-0.287</td>
<td>-0.479</td>
<td>-0.604</td>
<td>-0.715</td>
<td>-0.728</td>
</tr>
<tr>
<td>t-stat</td>
<td>-16.1</td>
<td>-14.5</td>
<td>-12.6</td>
<td>-9.3</td>
<td>-6.9</td>
</tr>
<tr>
<td>adj-R(^2)</td>
<td>0.27</td>
<td>0.24</td>
<td>0.20</td>
<td>0.14</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Panel B – Model simulations without realized disasters

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficient</td>
<td>-0.310</td>
<td>-0.519</td>
<td>-0.657</td>
<td>-0.789</td>
<td>-0.794</td>
</tr>
<tr>
<td>t-stat</td>
<td>-19.4</td>
<td>-16.7</td>
<td>-14.2</td>
<td>-10.5</td>
<td>-7.8</td>
</tr>
<tr>
<td>adj-R(^2)</td>
<td>0.34</td>
<td>0.28</td>
<td>0.24</td>
<td>0.17</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 5: Long-horizon predictability of excess stock returns.

The table reports the coefficient, t-statistics and adjusted-R\(^2\) of the regressions of cumulative excess returns over the horizons of 1, 2, 3, 5 and 7 years on the current log price-dividend ratio. Panel A reports the estimates of the full model and Panel B reports the estimates when disasters are turned off from the simulations. One thousand simulations are run at the monthly frequency over a 60-year horizon. Initial state variables are \(x_t = z_{1t} = z_{2t} = 0\), \(\lambda_t = \bar{\lambda}\), and \(\mu_t = \bar{\mu}\) and the calibration is provided in Table 3.

understand the timing of dividend growth risk and the shape of the term structures of equity return and interest rates, but are likely not sufficient to accurately explain the observed positive relation between the explanatory power of the price-dividend ratio and the horizon.

Since disasters are rare events, it is noteworthy to understand whether the model leads to stock-return predictability over time periods in which disasters do not realize. Panel B of Table 5 shows the regression results when disasters are turned off from the simulations. A strong negative intertemporal relationship between prices and future excess returns is still observable. This obtains because investors take into account the possibility that a disaster followed by a recovery could occur. Moreover, the probability of such an event is mean-reverting because the disaster intensity follows a square-root process.

5 Slow Disasters and the Term Structures of Equity

Nakamura, Steinsson, Barro, and Ursua (2013) document that rare disasters do not affect consumption instantaneously but gradually. Indeed, their estimation shows that disasters
take approximately four years to fully realize. The aim of this section is twofold. First, we model “slow disasters” in a parsimonious way that guarantees analytical tractability. Second, we show that the main message of the paper—disaster recovery leads to downward-sloping term structures of equity return—is robust to the property of “slow disasters”.

We model the joint dynamics of consumption and dividends as follows:

\[
\log C_t = x_t + s_{1t}, \\
\log D_t = x_t + s_{2t} + \log d_0, \quad d_0 \in (0, 1),
\]

with

\[
\begin{align*}
 dx_t &= (\mu - \frac{1}{2} \sigma_x^2 + \omega z_{1t}) dt + \sigma_x dW_{xt}, \\
 ds_{1t} &= \phi_s (z_{it} - s_{1t}) dt + \sigma_{s1} dW_{st}, \\
 ds_{2t} &= \phi_s (z_{it} - s_{2t}) dt + \sigma_{s2} dW_{st}, \\
 dz_{it} &= -\phi_z z_{it} dt + \alpha_i \xi_t dN_{zt},
\end{align*}
\]

where \( i \in \{1, 2\} \), \( \alpha_1 = 0 \), and \( \alpha_2 = \alpha \). Consumption and dividends are co-integrated because \( s_{1t} \) and \( s_{2t} \) are stationary; dividends are more sensitive than consumption to disasters and recoveries when \( \alpha > 1 \); disasters partially recover when \( \phi_z > 0 \) and \( \omega > 0 \); and the speed of reversion \( \phi_s \) governs the length of the disaster period. For simplicity, we turn off time-variation in expected growth \( \mu \) and disaster intensity \( \lambda \). Since the above dynamics belong to the affine class, we can again follow the methodology of Eraker and Shaliastovich (2008) and solve for prices as in the previous sections.

In this model, the parameters \( \lambda, \eta, \omega, \phi_z, \) and \( \phi_s \) characterize disasters and recoveries. We choose these parameters to match the probability of a disaster occurring, the size of a disaster, the length of the disaster period, the size of the recovery, and the length of the recovery period. The remaining parameters \( (\mu, \sigma_x, \sigma_{s1}, \sigma_{s2}, \alpha, d_0) \) are set to match the yearly mean and volatility of consumption and dividend growth, their 10-year variance ratios, and the mean and volatility of the dividend share of consumption. Empirical moments, theoretical moments, and parameter values are provided in Appendix A.6. The model fits the data well, which shows that accounting for “slow disasters” does not diminish the model’s ability to match the timing of fundamentals risk.

The upper left panel of Figure 9 depicts the deterministic evolution of log-consumption conditional on a disaster occurring at time 0 \((dN_{z0} = 1)\). It shows that our parsimonious dynamics are flexible enough to reproduce the typical path of consumption documented by Nakamura, Steinsson, Barro, and Ursua (2013).

The upper right panel of Figure 9 reports the term structures of consumption and dividend...
growth risk. Consistent with empirical findings, dividend growth risk decreases with the horizon, whereas consumption growth risk is almost constant.

We conclude our analysis by choosing preference parameters that guarantee downward-sloping term structures of equity return volatility and risk premia and reasonable values for the risk-free rate and equity premium. On the one hand, disaster recovery ($\phi_z > 0$) and levered exposition of dividends to disasters ($\alpha > 1$) imply that prices inherit the negative slope of the term structure of dividend growth risk for a large range of preference parameters. On the other hand, “slow disasters” imply that a high elasticity of intertemporal substitution is needed to generate sizeable equity premia. Indeed, slow disasters imply that consumption growth is safer than in our main model at short horizons. Therefore, similar equity premia
can only be obtained if the investor has a preference for early resolution of uncertainty. As in Nakamura, Steinsson, Barro, and Ursua (2013), we choose $\psi = 2$ and then set $\delta = 0.965$ and $\gamma = 8.5$. The lower left and right panels of Figure 9 show the term structures of equity return volatility and risk premia, respectively. The term structure of equity return volatility is downward-sloping for all horizons, whereas the term structure of risk premia is downward-sloping as long as the horizon is larger than about three years. The term structure of risk premia is upward-sloping in the very short term because disasters are not instantaneous in the present model.

The above choice of preference parameters leads to equity premia of about 4.24% and a risk-free rate of about 1.52%. Although these numbers do not perfectly match their empirical counterparts, they suggest that it is possible to reconcile disaster risk as a key driver of equity risk premia with the empirical evidence about the term structures of both fundamentals and equity return. This model, however, implies negative bond yields for most horizons, a downward-sloping term structure of interest rates for horizons ranging from zero to five years, and an upward-sloping term structure of interest rate for horizons larger than five years. To summarize, the observed negative slopes of term structures of equity return, the low risk-free rate, and high equity premia are robust to the property of “slow disasters”, but “slow disasters” imply a counter-factual term structure of interest rates.

6 Conclusion

Empirical research provides evidence that the term structures of equity return volatility and risk premia are downward-sloping. This finding is particularly important provided that leading asset-pricing models (Bansal and Yaron, 2004; Gabaix, 2012; Wachter, 2013) imply strongly upward-sloping term structures of equity return. In this paper, we show that the term structures of equity return obtained in a rare disasters model are downward-sloping when accounting for post-disaster recoveries—a feature of the data disregarded by most of the literature. In the presence of recoveries, disaster risk concentrates in the short term, which implies downward-sloping term structures of dividend growth risk, equity return volatility, and equity risk premia.

The model accounts for the observed slightly upward-sloping term structure of consumption growth risk (due to time-varying expected growth and disaster intensity) and the downward-sloping term structure of dividend growth risk (due to the levered exposition of dividends to disasters and recoveries). It reconciles high equity risk premia and a low risk-free rate with the shape of the term structures of equity return and interest rates for a broad range of realistic preference parameters.
References


A Appendix

A.1 Proof of Proposition 1

Following Eraker and Shaliastovich (2008), the state-price density satisfies

\[ d \log M_t = (\theta \log \delta - (\theta - 1) \log k_1 + (\theta - 1)(k_1 - 1)B'_c(Y_t - \mu_Y)dt - \Omega'dY_t, \]

where \( \mu_Y = (0, 0, 0, \bar{\lambda}, \bar{\mu})^\top \) and \( \Omega = \gamma(1, 1, 0, 0, 0)^\top + (1 - \theta)k_1B_c. \)

The coefficients \( A_c \in \mathbb{R}, B_c \in \mathbb{R}^5, \) and \( k_1 \in \mathbb{R} \) characterizing the wealth, \( V_t, \) and the state-price density, \( M_t, \) solve the following system of equations:

\[
\begin{align*}
0 &= \mathcal{K}^\top \chi - \theta(1 - k_1)B_c + \frac{1}{2} \chi^\top H\chi + l_1^\top (\phi(\chi) - 1) \\
0 &= \theta(\log \delta + k_0 - (1 - k_1)A_c) + \mathcal{M}^\top \chi + \frac{1}{2} \chi^\top h\chi + l_0^\top (\phi(\chi) - 1) \\
\theta \log k_1 &= \theta(\log \delta + (1 - k_1)B^\top_c \mu_Y) + \mathcal{M}^\top \chi + \frac{1}{2} \chi^\top h\chi + l_0^\top (\phi(\chi) - 1),
\end{align*}
\]

where \( \chi = \theta ((1 - \frac{1}{\psi})(1, 1, 0, 0, 0)^\top + k_1B_c) \), \( k_0 = -\log ((1 - k_1)^{1-k_1}k_1^k_1) \).

A.2 Proof of Proposition 2

Following Eraker and Shaliastovich (2008), the coefficient \( \Phi_0 \in \mathbb{R} \) and \( \Phi_1 \in \mathbb{R}^5 \) characterizing the risk-free rate \( r_t \) solve the following system of equations:

\[
\begin{align*}
\Phi_1 &= (1 - \theta)(k_1 - 1)B_c + \mathcal{K}^\top \Omega - \frac{1}{2} \Omega^\top H\Omega - l_1^\top (\phi(-\Omega) - 1) \\
\Phi_0 &= -\theta \log \delta + (\theta - 1)(\log k_1 + (k_1 - 1)B^\top_c \mu_Y) + \mathcal{M}^\top \Omega - \frac{1}{2} \Omega^\top h\Omega - l_0^\top (\phi(-\Omega) - 1).
\end{align*}
\]

A.3 Proof of Proposition 3

Following Eraker and Shaliastovich (2008), the deterministic functions \( a_j(\cdot) \in \mathbb{R} \) and \( b_j(\cdot) \in \mathbb{R}^5, j \in \{\text{con, div, } B\} \) characterizing the price of strips and non-defaultable bonds solve the following system of ordinary differential equations:

\[
\begin{align*}
b'_j(\tau) &= -\Phi_1 + \mathcal{K}^Q b_j(\tau) + \frac{1}{2} b_j(\tau)^\top Hb_j(\tau) + l_1^Q (\phi^Q(b_j(\tau)) - 1) \\
a'_j(\tau) &= -\Phi_0 + \mathcal{M}^Q b_j(\tau) + \frac{1}{2} b_j(\tau)^\top h_b_j(\tau) + l_0^Q (\phi^Q(b_j(\tau)) - 1)
\end{align*}
\]

subject to \( a_{\text{con}}(0) = 0, \ b_{\text{con}}(0) = (1, 1, 0, 0, 0)^\top, \ a_{\text{div}}(0) = 0, \ b_{\text{div}}(0) = (1, 0, 1, 0, 0)^\top, \ a_B(0) = 0, \) and \( b_B(0) = (0, 0, 0, 0, 0)^\top. \) The coefficients \( \mathcal{M}^Q \in \mathbb{R}^5, \mathcal{K}^Q \in \mathbb{R}^{5 \times 5}, \) \( l_0^Q \in \mathbb{R}^5, \) \( l_1^Q \in \mathbb{R}^{5 \times 5}, \) and
\( q^{Q}(.) \in \mathbb{R}^5 \) satisfy

\[
\begin{align*}
\mathcal{M}^Q &= \mathcal{M} - h\Omega \\
\ell^Q(Y_t) &= l(Y_t) \cdot g(-\Omega) \equiv \ell_0^Q + \ell_1^Q Y_t \\
\mathcal{K}^Q &= \mathcal{K} - H\Omega \\
g^Q(u) &= g(u - \Omega) / g(-\Omega).
\end{align*}
\]

\( \square \)

### A.4 Proof of Proposition 5

Following Duffie, Pan, and Singleton (2000), the deterministic functions \( \bar{a}_{CF}(.; .) \in \mathbb{R} \) and \( \bar{b}_{CF}(.; .) \in \mathbb{R}^5 \) characterizing the moment-generation function \( MGF(., .; .) \) solve the following system of ordinary differential equations:

\[
\begin{align*}
\bar{b}_{CF}'(\tau; u) &= \mathcal{K}^\top \bar{b}_{CF}(\tau; u) + \frac{1}{2} \bar{b}_{CF}(\tau; u)^\top H \bar{b}_{CF}(\tau; u) + l_1^\top \left( g(\bar{b}_{CF}(\tau; u)) - 1 \right) \\
\bar{a}_{CF}'(\tau; u) &= \mathcal{M}^\top \bar{b}_{CF}(\tau; u) + \frac{1}{2} \bar{b}_{CF}(\tau; u)^\top h \bar{b}_{CF}(\tau; u) + l_0^\top \left( g(\bar{b}_{CF}(\tau; u)) - 1 \right) ,
\end{align*}
\]

subject to \( a_{CF}(0) = 0, b_{CF}(0) = (u, u, 0, 0, 0)^\top \) if \( CF = \text{con} \), and \( b_{CF}(0) = (u, 0, u, 0, 0)^\top \) if \( CF = \text{div} \).

\( \square \)

### A.5 Accuracy of the Approximation

In this section, we investigate whether disasters (i.e. jumps in consumption) affect the accuracy of the log-linear approximation of the wealth return dynamics. To do so, we note that the sum of discounted (with the state-price density \( M_t \)) wealth and cumulative discounted past consumption is a martingale, \( G_t \), under the objective probability measure. That is,

\[
G_t \equiv M_t V_t + \int_0^t M_s C_s ds
\]

satisfies

\[
0 = \frac{1}{dt} \mathbb{E}_t [dG_t] = \frac{1}{dt} \mathbb{E}_t [d(M_t V_t)] + M_t C_t.
\]

Substituting

\[
V_t = C_t e^{v_c(z_{1t}, \lambda_t, \mu_t)}
\]

in Equation (3) yields a partial differential equation (PDE) for the log wealth-consumption ratio \( v_c(z_{1t}, \lambda_t, \mu_t) \). Proposition 1 states that the log wealth-consumption ratio is approximated as follows: \(^\text{18}\)

\[
v_c(z_{1t}, \lambda_t, \mu_t) \approx A_c + B_c^\top Y = A_c + B_{c,2} z_{1t} + B_{c,4} \lambda_t + B_{c,5} \mu_t.
\]

Therefore, substituting Equation (4) in the right-hand side of Equation (3) yields PDE residuals that are different from zero unless the approximation is in fact the true solution of the

\(^{18}\)The loadings \( B_{c,1} \) and \( B_{c,3} \) on \( x_t \) and \( z_{2t} \) are indeed equal to zero.
Figure 10: Effect of disasters on the approximation accuracy.

The left, middle, and right panels depict the PDE residuals (as a fraction of $M_t V_t$) as a function of $\lambda_t$, $\mu_t$, and $z_{1t}$, respectively. The solid blue and dashed red lines are obtained with the baseline calibration and without disasters ($\eta \to \infty$), respectively. The range for $\lambda_t$ and $\mu_t$ is the corresponding 99% confidence interval, whereas that for $z_{1t}$ is $(E(\xi_t), 0)$. Unless stated otherwise, state variables are $x_t = z_{1t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$ and the calibration is provided in Table 3.

PDE.

Figure 10 depicts the PDE residuals (as a fraction of $M_t V_t$) over the state space. The solid blue line is obtained by setting the parameters of the model to their baseline values provided in Table 3. The dashed red line differs from the solid blue line in one respect, namely the size of the disaster is set to zero ($\eta \to \infty$). The residuals obtained with our baseline calibration and without disasters are of the same order of magnitude, showing that the log-linear approximation remains accurate in the presence of disasters.\footnote{Campbell, Lo, and MacKinlay (1997) and Bansal, Kiku, and Yaron (2012) document that the log-linear approximation is highly accurate when shocks are normally distributed.}

A.6 Slow Disasters and the Term Structures of Equity

Table 6 reports the parameter values of the slow disasters model described in Section 5. The parameters are calibrated to match the empirical moments of consumption and dividends provided in the second column of Table 7. The corresponding theoretical moments are provided in the third column of Table 7.
Table 6: Calibration of the slow disasters model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent shock:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td>$\sigma_x$</td>
<td>0</td>
</tr>
<tr>
<td>Long-run expected growth</td>
<td>$\mu$</td>
<td>0.032</td>
</tr>
</tbody>
</table>

| Disasters:                         |        |       |
| Long-term jump intensity           | $\lambda$ | 0.028 |
| Jump size parameter               | $\eta$ | 2     |
| Permanent jump size parameter     | $\omega$ | 0.05  |
| Speed of recovery                  | $\phi_z$ | 0.33  |
| Speed of disaster realization      | $\phi_s$ | 0.33  |
| Transitory noise in consumption growth | $\sigma_{s1}$ | 0.037 |
| Transitory noise in dividend growth | $\sigma_{s2}$ | 0.17  |

| Dividends:                         |        |       |
| Leverage parameter                 | $\alpha$ | 2     |
| Steady state dividend share        | $d_0$  | 0.05  |

| Preferences:                      |        |       |
| Relative risk aversion            | $\gamma$ | 8.5   |
| Elasticity of intertemporal substitution (EIS) | $\psi$ | 2     |
| Time discount factor              | $\delta$ | 0.965 |

Table 7: Empirical and theoretical moments of consumption and dividends.

Empirical moments are computed using consumption and dividend data on the sample 1929-2013 at yearly frequency from NIPA tables 1.1.6, 1.1.10 and 1.11. Theoretical moments are computed using the slow disasters model described in Section 5.